Optical flow
Today

From images to video

- Introduction
- Definitions and Key Assumptions
- Lucas-Kanade
- Applications
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From images to video

- A video is a sequence of frames captured over time
- Now our image data is a function of space \((x,y)\) and time \((t)\)
Motion estimation techniques

- **Optical flow**
  - Compute image motion at each and every pixel from spatio-temporal image brightness variations

- **Feature tracking**
  - Extract visual features and “track” them over multiple frames
Optical flow

- Compute motion for all pixels
Feature tracking

- Track only “good” features
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Optical flow - definition

- The optical flow is the *apparent* motion of brightness patterns in the image.

Pierre Kornprobst's Demo
Examples of Motion fields

- **Forward motion**
- **Rotation**
- **Horizontal translation**
- **Closer objects appear to move faster!!**
Motion Field & Optical Flow Field

- Underlying assumption:
The apparent motion field is a projection of the real 3D motion onto the 2d image

\[ \mathbf{u} = (u, v) \]

CCD

3D motion vector

2D optical flow vector

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When does it break?

The screen is stationary yet displays motion.

Homogeneous objects generate zero optical flow.

Fixed sphere. Changing light source.

Non-rigid texture motion.
The Optical Flow Field

Still, in many cases it does work....

Goal:
Find for each pixel a velocity vector \( \vec{u} = (u, v) \) which says:
- In which direction it is moving
- How quickly is the pixel moving across the image
How do we actually do that?
Estimating Optical Flow

- **Assumption 1**: Brightness constancy
- The image intensity $I$ is constant

\[
I(x, y, t) = I(x + dx, y + dy, t + dt)
\]
Estimating Optical Flow

- The brightness constancy equation

\[ I(x, y, t) = I(x + dx, y + dy, t + dt) \]
Estimating Optical Flow

- The brightness constancy equation

\[ I(x, y, t) = I(x + dx, y + dy, t + dt) \]

- Assumption 2: Motion is small

First order Taylor expansion:

\[
I(x + dx, y + dy, t + dt) = I(x, y, t) + \frac{\partial I}{\partial x} dx + \frac{\partial I}{\partial y} dy + \frac{\partial I}{\partial t} dt
\]

\[
0 = \frac{\partial I}{\partial x} dx + \frac{\partial I}{\partial y} dy + \frac{\partial I}{\partial t} dt
\]
Estimating Optical Flow

- Simplify notations: \( I_x dx + I_y dy + I_t dt = 0 \)
- Divide by \( dt \) and denote \( u = \frac{dx}{dt}, v = \frac{dy}{dt} \)
- Final equation is: \( I_x u + I_y v = -I_t \)
Estimating Optical Flow

- Can we use this equation to recover image motion at each pixel?

\[ I_x u + I_y v = \nabla I \begin{bmatrix} u \\ v \end{bmatrix} = -I_t \]

- **Problem**
  - 1 equation per pixel, 2 unknowns
  - This means we cannot recover the motion component perpendicular to the gradient

This leads to the Aperture Problem...
The aperture problem
The aperture problem
The aperture problem

- For points on a line of fixed intensity we can only recover the normal flow.

Where did the blue point move to?

We need additional constraints.
Solving the ambiguity

Sometimes enlarging the aperture can help
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Solving the ambiguity

- **Assumption 3: Spatial coherence [Lucas & Kanade 1981]**: Assume constant \((u, v)\) in small neighborhood

\[
I_x u + I_y v = -I_t \quad \rightarrow \quad \begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -I_t
\]

\[
\begin{bmatrix}
I_{x1} & I_{y1} \\
I_{x2} & I_{y2} \\
\vdots & \vdots \\
I_{xL} & I_{yL}
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix}
= -
\begin{bmatrix}
I_{t1} \\
I_{t2} \\
\vdots \\
I_{tL}
\end{bmatrix}
\]

\[A\hat{u} = b\]
Goal: Minimize \( \| A\tilde{u} - b \|^2 \)

Method: Least-Squares

\[ A\tilde{u} = b \]

\[ A^T A \tilde{u} = A^T b \]

\[ \tilde{u} = \left( A^T A \right)^{-1} A^T b \]
How does Lucas-Kanade behave?

\[ \vec{u} = \left( A^T A \right)^{-1} A^T b \]

\[ A^T A = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \]

We want this matrix to be invertible

\( \rightarrow \) no zero eigenvalues
How does Lucas-Kanade behave?

- Edge $\Rightarrow A^T A$ becomes singular

\[
\begin{bmatrix}
-I_y, I_x \\
I_x, I_y
\end{bmatrix}
\quad \begin{bmatrix}
(I_x, I_y)
\end{bmatrix}
\]

\[
\begin{bmatrix}
\sum I_x^2 & \sum I_x I_y \\
\sum I_x I_y & \sum I_y^2
\end{bmatrix}
\begin{bmatrix}
-I_y \\
I_x
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
-I_y \\
I_x
\end{bmatrix}
\text{is eigenvector with eigenvalue 0}
\]
How does Lucas-Kanade behave?

- Homogeneous $\Rightarrow A^T A \approx 0 \Rightarrow 0$ eigenvalues

$\begin{pmatrix} I_x, I_y \end{pmatrix} \approx 0$
How does Lucas-Kanade behave?

- Textured regions $\Rightarrow$ two high eigenvalues

$$(I_x, I_y) \neq 0$$
How does Lucas-Kanade behave?

- **"Corner"**
  - $\lambda_1$ and $\lambda_2$ are large,
  - $\lambda_1 \sim \lambda_2$

- **"Edge"**
  - $\lambda_1 \gg \lambda_2$

- **"Flat" region**
  - $\lambda_1$ and $\lambda_2$ are small

- **"Edge"**
  - $\lambda_2 \gg \lambda_1$
How does Lucas-Kanade behave?

Edge $\implies A^T A$ becomes singular

Homogeneous regions $\implies$ low gradients $A^T A \approx 0$

High texture $\implies$
When assumptions break

- Brightness constancy is **not** satisfied
  - Correlation based methods
    (or any other descriptor)

- A point does **not** move like its neighbors
  - what is the ideal window size?
    - Regularization based methods

- The motion is **not** small (Taylor expansion doesn’t hold)
  - Use multi-scale estimation
Multi-Scale Flow Estimation

image $I_t$

Gaussian pyramid of image $I_t$

$u=10 \text{ pixels}$

$u=5 \text{ pixels}$

$u=2.5 \text{ pixels}$

$u=1.25 \text{ pixels}$

image $I_{t+1}$

Gaussian pyramid of image $I_{t+1}$

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Multi-Scale Flow Estimation

- Gaussian pyramid of image $I_t$
- run Lucas-Kanade
- warp & upsample
- Gaussian pyramid of image $I_{t+1}$
- run Lucas-Kanade
Example
Multi-resolution registration
When assumptions break

- Brightness constancy is **not** satisfied
  - Correlation based methods (or any other descriptor)

- A point does **not** move like its neighbors
  - what is the ideal window size?
  - Regularization based methods

- The motion is **not** small (Taylor expansion doesn’t hold)
- Aliasing
  - Use multi-scale estimation
When assumptions break

- Brightness constancy is **not** satisfied
  - Correlation based methods
    (or any other descriptor)

- A point does **not** move like its neighbors
  - what is the ideal window size?
  - Regularization based methods

- The motion is **not** small (Taylor expansion doesn’t hold)
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Autonomous Cars

https://www.youtube.com/watch?v=rB1BmBOkKTw
Segmentation by Flow

https://www.youtube.com/watch?v=QwmBSTWgr_s

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Volumetric Reconstruction from Flow

https://www.youtube.com/watch?v=lk_yX-O_Y5c
Single image animation

https://youtu.be/-RetOjLIFhw
End – Optical flow

Now you know how it works