About this lecture

In this lecture we introduce INFORMALLY the most important operations based on morphology, just to give you the intuitive feeling.
What We Do Today.

- **Introduction to** Morphological Operators
  - Used generally on binary images, e.g., background subtraction results!

- Good for, e.g.,
  - Noise removal in background
  - Removal of holes in foreground / background
EROSION
A first Example: Erosion

- **Erosion** is an important morphological operation
- Applied **Structuring Element**:

```
1 1 1
1 1 1
1 1 1
```

Set of coordinate points =

\{ (-1, -1), (0, -1), (1, -1),
(-1, 0), (0, 0), (1, 0),
(-1, 1), (0, 1), (1, 1) \}
EROSION

\[ A \Theta B = \text{AND}[A, B] \]

A-image

B- Structuring element
Example for Erosion

Input image:

```
1 0 0 0 1 1 1 1 0 1 1 1
```

Structuring Element:

```
1 1 1 1
```

Output Image:

```
0
```
Example for Erosion

Input image:

```
1 0 0 0 1 1 1 1 0 1 1 1
```

Structuring Element:

```
1 1 1
```

Output Image:

```
0 0
```
Example for Erosion

Input image

| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |

Structuring Element

| 1 | 1 | 1 | 1 |

Output Image

| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
Example for Erosion

Input image

Structuring Element

Output Image
Example for Erosion

Input image
1 0 0 0 1 1 1 1 0 1 1

Structuring Element
1 1 1 1

Output Image
0 0 0 0 0 1
Example for Erosion

Input image:

```
1 0 0 0 1 1 1 1 0 1 1 1
```

Structuring Element:

```
1 1 1 1
```

Output Image:

```
0 0 0 0 0 1 0
```
Example for Erosion

Input image

Structuring Element

Output Image
Example for Erosion

**Input image**

```
1 0 0 0 1 1 1 1 0 1 1 1
```

**Structuring Element**

```
1 1 1 1
```

**Output Image**

```
0 0 0 0 0 1 0 0 0 0
```
Introduction

- Structuring Element
- Erosion
- Dilation
- Opening
- Closing
- Outlook: Hit-and-miss Operation, Thinning, Thickening
Structuring Element (Kernel)

- Structuring Elements can have varying sizes
- Usually, element values are 0, 1
- Structural Elements have an origin
- For thinning, other values are possible
- Empty spots in the Structuring Elements are *don’t care’s!*

```
Box

Disc
```
Dilation & Erosion

- Basic operations
- Are dual to each other:
  - Erosion shrinks foreground, enlarges Background
  - Dilation enlarges foreground, shrinks background
Erosion

- **Erosion** is the set of all points in the image, where the structuring element “fits into”.
- Consider each foreground pixel in the input image
  - If the structuring element fits in, write a “1” at the origin of the structuring element!
- Simple application of **pattern matching**
- **Input:**
  - Binary Image (Gray value)
  - Structuring Element, containing only 1s!
Another example of erosion

- White = 0, black = 1, dual property, image as a result of erosion gets darker
Counting Coins

- Counting coins is difficult because they touch each other!
- Solution: Binarization and Erosion separates them!
DILATION
Example: Dilation

- **Dilation** is an important morphological operation.

  - Applied **Structuring Element**:

    ![Structuring Element Diagram]

    Set of coordinate points =
    
    \[
    \{ (-1, -1), (0, -1), (1, -1), \\
    (-1, 0), (0, 0), (1, 0), \\
    (-1, 1), (0, 1), (1, 1) \} 
    \]}
DILATION

\[ A\Theta B = \text{OR}[A,B] \]

A - image
B - Structuring element
Dilation

- **Dilation** is the set of all points in the image, where the structuring element “touches” the foreground.

- Consider each pixel in the input image
  - If the structuring element touches the foreground image, write a “1” at the origin of the structuring element!

- **Input:**
  - Binary Image
  - Structuring Element, containing only 1s!!
Example for Dilation

Input image: 1 0 0 0 1 1 1 1 0 1 1 1

Structuring Element: 1 1 1 1

Output Image: 1
Example for Dilation

Input image

1 0 0 0 1 1 1 1 0 1 1 1

Structuring Element

1 1 1

Output Image

1 0
Example for Dilation

Input image

1 0 0 0 1 1 1 1 0 1 1 1

Structuring Element

1 1 1 1

Output Image

1 0 1
Example for Dilation

Input image

| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |

Structuring Element

| 1 | 1 | 1 |

Output Image

| 1 | 0 | 1 | 1 |
Example for Dilation

Input image

| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |

Structuring Element

| 1 | 1 | 1 | 1 |

Output Image

| 1 | 0 | 1 | 1 | 1 | 1 |
Example for Dilation

Input image: 
1 0 0 0 1 1 1 1 0 1 1

Structuring Element: 
1 1 1

Output Image: 
1 0 1 1 1 1 1
Example for Dilation

Input image

1 0 0 0 1 1 1 1 0 1 1 1

Structuring Element

1 1 1

Output Image

1 0 1 1 1 1 1 1 1
Example for Dilation

Input image

| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |

Structuring Element

| 1 | 1 | 1 | 1 |

Output Image

| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
Another Dilation Example

- Image get lighter, more uniform intensity
Edge Detection

1. Dilate input image
2. Subtract input image from dilated image
3. Edges remain!
Opening & Closing

- Important operations
- Derived from the fundamental operations
  - Dilatation
  - Erosion
- Usually applied to binary images, but gray value images are also possible
- Opening and closing are dual operations
OPENING
Opening

- Similar to Erosion
  - Spot and noise removal
  - Less destructive

- Erosion next dilation

- the same structuring element for both operations.

Input:
- Binary Image
- Structuring Element, containing only 1s!
Opening

- Take the structuring element (SE) and slide it around inside each foreground region.
  - All pixels which can be covered by the SE with the SE being entirely within the foreground region will be preserved.
  - All foreground pixels which can not be reached by the structuring element without lapping over the edge of the foreground object will be eroded away!
- Opening is idempotent: Repeated application has no further effects!
Opening

- Structuring element: 3x3 square
Opening Example

- Opening with a 11 pixel diameter disc
Opening Example

- 3x9 and 9x3 Structuring Element
Opening on Gray Value Images

- 5x5 square structuring element
Use Opening for Separating Blobs

- Use large structuring element that fits into the big blobs
- Structuring Element: 11 pixel disc
CLOSING
Closing

- Similar to Dilation
  - Removal of holes
  - Tends to enlarge regions, shrink background

- Closing is defined as a Dilatation, followed by an Erosion using the same structuring element for both operations.

- Dilation next erosion!

- Input:
  - Binary Image
  - Structuring Element, containing only 1s!
Closing

- Take the structuring element (SE) and slide it around outside each foreground region.
  - All background pixels which can be covered by the SE with the SE being entirely within the background region will be preserved.
  - All background pixels which can not be reached by the structuring element without lapping over the edge of the foreground object will be turned into a foreground.

- Opening is idempotent: Repeated application has no further effects!
Closing

- Structuring element: 3x3 square
Closing Example

- Closing operation with a 22 pixel disc
- Closes small holes in the foreground
Closing Example 1

1. Threshold
2. Closing with disc of size 20
Closing Example 2

- Good for further processing: E.g. Skeleton operation looks better for closed image!

![skeleton of Thresholded and next closed](image)
Closing Gray Value Images

- 5x5 square structuring element
Opening & Closing

- Opening is the *dual* of closing
- i.e. *opening the foreground* pixels with a particular structuring element
- is equivalent to *closing the background* pixels with the same element.
HIT and MISS
Hit-and-miss Transform

- Used to look for particular patterns of foreground and background pixels
- Very simple object recognition
- All other morphological operations can be derived from it!!

Input:
  - Binary Image
  - Structuring Element, containing 0s and 1s!!
Hit-and-miss Transform

- Example for a Hit-and-miss Structuring Element
- Contains 0s, 1s and *don’t care’s*.
- Usually a “1” at the origin!
Hit-and-miss Transform

- Similar to Pattern Matching:
- If foreground and background pixels in the structuring element exactly match foreground and background pixels in the image, then the pixel underneath the origin of the structuring element is set to the foreground color.
Corner Detection with Hit-and-miss Transform

- Structuring Elements representing four corners

![Structuring Elements](image)
Corner Detection with Hit-and-miss Transform

- Apply each Structuring Element
- Use OR operation to combine the four results
Basic THINNING
Thinning

1. Used to remove selected foreground pixels from binary images.

2. After edge detection, lines are often thicker than one pixel.

3. Thinning can be used to thin those lines to one pixel width.
Definition of Thinning

- Let $K$ be a kernel and $I$ be an image

$$\text{thin}(I, K) = I - \text{HitAndMiss}$$

with $0-1=0$!

- If foreground and background fit the structuring element exactly, then the pixel at the origin of the SE is set to 0

- Note that the value of the SE at the origin is 1 or don’t care!
Example Thinning

We use two Hit-and-miss Transforms
Basic
THICKENING
Thickening

- Used to grow selected regions of foreground pixels
- E.g. applications like approximation of convex hull
Definition Thickening

- Let $K$ be a kernel and $I$ be an image

$$\text{thicken}(I, K) = I + \text{HitAndMiss}$$

with $1+1=1$

- If foreground and background match exactly the SE, then set the pixel at its origin to $1$!

- Note that the value of the SE at the origin is 0 or don’t care!
Example Thickening

\[
\begin{array}{c|c}
1 & 1 \\
1 & 0 \\
1 & 0 \\
\end{array}
\quad \begin{array}{c|c}
1 & 1 \\
0 & 1 \\
0 & 1 \\
\end{array}
\]