Exercise 2

1. Given a game $G$, with a finite set of players $N$, and a finite set of actions $A_i$ for each player, a profile of actions $a \in A = \prod_{i=1}^{n} A_i$ is called a strong equilibrium if $\forall B \subseteq N$, and $\forall s_B \in \prod_{j \in B} A_i$, $\exists l \in B$ such that $u_l(a_{-B}, s_B) \leq u_l(a)$.

A network traffic game is a parallel-links congestion game, if in the graph $G = (V, E)$, $E = \cup_j P_j$ where any $P_j$ is a path from $s$ to $t$, and $P_p \cap P_q = \emptyset$ for any $p \neq q$.

Prove: all pure Nash equilibria of a parallel links game are strong equilibria.

2. Given a game $G$, with a finite set of players $N$, and a finite set of actions $A_i$ for each player, where $\Delta(A_i)$ are the mixed strategies for player $i$, and $\Delta(A) = \prod_{i=1}^{n} \Delta(A_i)$. Let $u_i(b)$ be the expected utility of $i$ given a profile of mixed strategies $b \in \Delta(A)$. A profile of pure strategies $a \in A = \prod_{i=1}^{n} A_i$ is called a mixed-strong equilibrium if $\forall B \subseteq N$, and $\forall s_B \in \prod_{j \in B} \Delta(A_j)$, $\exists l \in B$ such that $u_l(a_{-B}, s_B) \leq u_l(a)$.

A resource selection game is defined as follows. Consider a set of resources $R = \{r_1, \ldots, r_m\}$ and let $N = \{1, 2, \ldots, n\}$ be a set of players. Each agent’s (pure) strategy is to choose a single resource from $R$. The payoff of player $i$ who chooses resource $j$, where the total number of players choosing $j$ is $k$ is $-c_j(k)$, where $c_j(0) = 0$ and $c_j$ is monotonic increasing (for every $j$). Prove of disprove: any resource selection game has a mixed-strong equilibrium of pure strategies.

3. Exercises from book (not for delivery): chapter 8 (1,2,3,4)