MULTICUT ON TREES:
Let $G = (V, E)$ be a graph equipped with non-negative edge weights $c : E \rightarrow \mathbb{R}_+$ and let $\{(s_i, t_i)\}_{i=1}^k$ be $k$ pairs of special vertices. A collection of edges $M \subseteq E$ is said to disconnect $\{(s_i, t_i)\}_{i=1}^k$ if $(V, E \setminus M)$ does not contain any path between $s_i$ and $t_i$ for all $i = 1, \ldots, k$. The goal in the MULTICUT problem is to find a collection of edges $M \subseteq E$ that disconnects $\{(s_i, t_i)\}_{i=1}^k$ and is of minimum total weight. We will focus on the case where $G$ is a tree.

1. Formulate a linear programming relaxation for the problem. (20 points)
   (Hint: use covering constraints for each relevant path in the input tree)

2. Formulate the dual of the relaxation. (20 points)

3. Present a primal-dual algorithm achieving an approximation guarantee of 2. (60 points)
   (Hint: the dual variable increased in each step should correspond to the pair $(s_i, t_i)$ whose least common ancestor is the deepest in the tree, and preform a cleanup step in reverse order of edge choice.)