Transactional Information Systems:

Theory, Algorithms, and the Practice of Concurrency Control and Recovery

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“Teamwork is essential. It allows you to blame someone else.” (Anonymous)
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“No matter how complicated a problem is, it usually can be reduced to a simple comprehensible form which is often the best solution” (An Wang)

“Every problem has a simple, easy-to-understand, wrong answer.” (Anonymous)
Definition 2.3 (Object Model Transaction): A transaction $t$ is a (finite) tree of labeled nodes with
- the transaction identifier as the label of the root node,
- the names and parameters of invoked operations as labels of inner nodes, and
- page-model read/write operations as labels of leaf nodes, along with a partial order $<$ on the leaf nodes such that for all leaf-node operations $p$ and $q$ with $p$ of the form $w(x)$ and $q$ of the form $r(x)$ or $w(x)$ or vice versa, we have $p < q \lor q < p$.

Special case: layered transactions (all leaves have same distance from root)

Derived inner-node ordering: $a < b$ if all leaf-node descendants of $a$ precede all leaf-node descendants of $b$. 
Example: DBS Internal Layers

\[
\begin{align*}
&t \\
&\downarrow 1 \\
&\text{Search ("Austin"')} \\
&\downarrow r (r) \quad r (l) \\
&\text{Fetch(x)} \quad \text{Fetch(y)} \\
&\downarrow r (p) \quad r (q) \\
&\text{Store(z)} \\
&\downarrow r (f) \quad r (p) \quad w (p) \quad r (r) \quad r (l) \quad w (l)
\end{align*}
\]
Example: Business Objects

Account x

Withdraw (x, 1000)

Append (h, ...)

Search (...)
Fetch (x)
Modify (x)
Fetch (a)
Fetch (d)
Store (e)
Modify (d)
Modify (a)

Search (...)
Fetch (y)
Modify (y)

Deposit (y, 1000)
Object-Model Schedules

Definition 6.1 (Object Model History):
For transaction trees \( \{t_1, ..., t_n\} \) a **history** \( s \) is a **partially ordered forest** \((op(s), <_s)\) with node set \( op(s) \) and partial order \( <_s \) of leaves such that

- \( op(s) \subseteq \bigcup_{i=1..n} op_i \cup \bigcup_{i=1..n} \{c_i, a_i\} \) and \( \bigcup_{i=1..n} op_i \subseteq op(s) \)
- for all \( t_i; c_i \in op(s) \iff a_i \not\in op(s) \)
- \( a_i \) or \( c_i \) is a leaf node with \( t_i \) as parent
- \( \bigcup_{i=1..n} <_i \subseteq <_s \)
- for all \( t_i \) and for all \( p \in op_i; p <_s a_i \) or \( p <_s c_i \)
- for all leaves \( p, q \) that access the same data item with \( p \) or \( q \) being a write:
  - either \( p <_s q \) or \( q <_s p \)

Definition 6.2 (Tree Consistent Node Ordering):
In history \( s = (op(s), <_s) \) the leaf ordering \( <_s \) is extended to arbitrary nodes:
\( p <_s q \) if for all leaf-level descendants \( p' \) of \( p \) and \( q' \) of \( q \):
\( p' <_s q' \).

Definition 6.3 (Object Model Schedule):
A **prefix** of history \( s = (op(s), <_s) \) is a forest \( s' \) \((op(s'), <'_s)\) with \( op(s') \subseteq op(s) \) and \( <'_s \subseteq <_s \) s.t. for each \( p \in op(s') \) all ancestors of \( p \) and all nodes \( q \) with \( q <_s p \) are in \( op(s') \) and \( <'_s \) equals \( <_s \) when restricted to \( op(s') \).

An **object model schedule** is a prefix of an object model history.
Example: Object-Model Schedule

Notation:
withdraw_{11}(a) \ withdraw_{21}(b) \ deposit_{22}(c) \ ...
\ r_{111}(p) \ r_{211}(q) \ w_{112}(p) \ w_{113}(t) \ w_{212}(q) \ w_{213}(t) \ r_{221}(r) \ w_{222}(r) \ ...
Definition 6.4 (Serial Object Model Schedule):
An object model schedule is **serial** if its roots $t_1, ..., t_n$ are **totally ordered** and for each $t_j$ and each $i > 0$ the descendants with distance $i$ from $t_j$ are totally ordered.

Definition 6.5 (Isolated Subtree):
A node $p$ and the corresponding subtree in a schedule are called **isolated** if
- for all nodes $q$ other than ancestors or descendants of $p$ the property holds that for all leaves $w$ of $q$ either $w < p$ or $p < w$
- for each $i > 0$ the descendants of $p$ with distance $i$ from $p$ are totally ordered

Definition 6.6 (Layered History and Schedule):
An object model history is **layered** if all leaves other than $c$ or $a$ have identical distance from their roots; for leaf-to-root distance $n$ this is called an **$n$-level history**. Operations with distance $i$ from the leaves are called **level-$i$ ($L_i$) operations**. A **layered schedule** is a prefix of a layered history.
Examples of Non-layered Schedules

non-layered

non-layered

Isolated sub-tree of deposit(c)
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Flat Object Schedules

**Definition 6.7 (Flat Object Schedule):**
A 2-level schedule $s$ is called **flat** if for each $p$, $q$ of $L_1$ operations:

- for all $p' \in \text{child}(p)$ and all $q' \in \text{child}(q)$: $p' <_s q'$ or
- for all $p' \in \text{child}(p)$ and all $q' \in \text{child}(q)$: $q' <_s p'$, and
- for all $p'$, $p'' \in \text{child}(p)$: $p' <_s p''$ or $p'' <_s p'$

**Definition 6.8 ((State-independent) Commutative Operations):**
Operations $p$ and $q$ are **commutative** if for all possible sequences of operations $\alpha$ and $\omega$ the return parameters in the sequence $\alpha p q \omega$ are identical to those in $\alpha q p \omega$. 
Example: Flat Object Schedule

(State-independent)
Commutativity table:

<table>
<thead>
<tr>
<th></th>
<th>withdraw (x,Δ₂)</th>
<th>deposit (x,Δ₂)</th>
<th>getbalance (x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>withdraw (x,Δ₁)</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>deposit (x,Δ₁)</td>
<td>—</td>
<td>+</td>
<td>—</td>
</tr>
<tr>
<td>getbalance (x)</td>
<td>—</td>
<td>—</td>
<td>+</td>
</tr>
</tbody>
</table>
Commutativity-based Reducibility

**Definition 6.9 (Commutativity Based Reducibility):**
A flat object schedule $s$ is **commutativity based reducible** if it can be transformed into a serial schedule by applying the following rules:

- **Commutativity rule:**
  the order of ordered operations $p, q$, say $p <_s q$, can be reversed if
  - both are isolated, adjacent, and commutative and
  - the operations belong to different transactions.

- **Ordering rule:**
  Unordered leaf operations $p, q$ can be arbitrarily ordered if they are commutative.

**Definition 6.10 (Conflict Equivalence and Conflict Serializability):**
Two flat object schedules $s$ and $s'$ are **conflict equivalent** if they consist of the same operations and have the same ordering for all non-commutative pairs of $L_1$ operations.
$s$ is **conflict serializable** if it is conflict equivalent to a serial schedule.

**Theorem 6.1:**
For a flat object schedule $s$ the following three conditions are equivalent:
$s$ is conflict serializable, $s$ has an acyclic conflict graph,
$s$ is commutativity-based reducible.
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Example: Layered Object Schedule with Non-isolated Subtrees

We can push the fetch(x) tree before the store(z) tree.

We get:
fetch(x) store(z) modify(y) modify(w) modify(y)
Same effect as:
store(z) modify(y) modify(w) fetch(x) modify(y)

We can push the modify(y) tree after the modify(w) tree.
Definition 6.11 (Tree Reducibility):
Object-model history $s = (op(s), <_s)$ is tree reducible if it can be transformed into a total order of its roots by applying the following rules:

- **Commutativity rule:**
  the order of ordered leaf operations $p, q$, say $p <_s q$, can be reversed if
  - both are isolated, adjacent, and commutative, and
  - the operations belong to different transactions, and
  - $p$ and $q$ do not have ancestors, $p'$ and $q'$, that are non-commutative and totally ordered in the order $p' <_s q'$.

- **Ordering rule:**
  Unordered leaf operations $p, q$ can be arbitrarily ordered if they are commutative.

- **Tree pruning rule:**
  An isolated subtree can be replaced by its root.

An object-model schedule is tree reducible if its committed projection is tree reducible.
Example: Reducible Layered Object Schedule with Non-isolated Subtrees

\[ t_1 \quad t_2 \]

\[
\begin{align*}
\text{store}(z) & \quad \text{fetch}(x) \\
r(t) & \quad r(p) \quad r(q) \\
r(t) & \quad r(p) \quad w(q) \quad w(p) \quad w(t) \\
r(t) & \quad r(p) \quad w(p) \\
r(t) & \quad r(p) \quad w(p) \quad w(t) \\
r(q) & \quad w(q) \quad w(p) \quad w(t) \\
\text{modify}(y) & \quad \text{modify}(y) \quad \text{modify}(w) \\
r(t) & \quad r(p) \quad w(p) \quad w(t) \\
r(t) & \quad r(p) \quad w(p) \quad w(t) \\
\end{align*}
\]

\[ t_1 < t_2 \]
Example: Non-reducible Layered Object Schedule

\[
\begin{align*}
t_1 & \quad \text{store}(z) \\
r(t) & \quad r(p) \quad r(q) \quad w(q) \quad w(p) \\
t_2 & \quad \text{fetch}(x) \\
r(t) & \quad r(p) \quad w(t)
\end{align*}
\]
Example: Reducible Non-layered Object Schedule

Conflicting operation pairs:
<Payment, Payment>, <Append, Append>, <r, w>, <w, r>, <w, w>

Observe that Payments are not totally ordered
Example: Reducible Non-layered Object Schedule

Conflicting operation pairs:
\(<\text{Payment, Payment}>, <\text{Append, Append}>, <\text{r, w}>, <\text{w, r}>, <\text{w, w}>\)
Example: Reducible Non-layered Object Schedule

Conflicting operation pairs:
<Payment, Payment>, <Append, Append>, <r, w>, <w, r>, <w, w>
Example: Reducible Non-layered Object Schedule

Conflicting operation pairs:
<Payment, Payment>, <Append, Append>, <r, w>, <w, r>, <w, w>
Example: Reducible Non-layered Object Schedule

Conflicting operation pairs:
<Payment, Payment>, <Append, Append>, <r, w>, <w, r>, <w, w>
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Sufficient Conditions for Tree Reducibility

**Definition 6.13 (Level-to-Level Schedule):**
For an n-level schedule $s = (\text{op}(s), <_s)$ with layers $L_0, ..., L_n$, the **level-to-level schedule from $L_i$ to $L_{(i-1)}$, or $L_i$-to-$L_{(i-1)}$ schedule**, is a conventional 2-level schedule $s' = (\text{op}(s'), <_{s'})$ with
- $\text{op}(s')$ consisting of the $L_{(i-1)}$ operations of $s$,
- $<_{s'}$ being the restriction of the extended order $<_s$ to the $L_{(i-1)}$ operations,
- $L_i$ operations of $s$ as roots, and
- the same parent-child relationship as in $s$.

**Theorem 6.2:**
Let $s$ be an n-level schedule. If for each $i$, $0 < i \leq n$, the $L_i$-to-$L_{(i-1)}$ schedule derived from $s$ is in OCSR, then $s$ is tree-reducible.

Observe that CSR requires conflicting operations to be ordered which implies an ordering on their leaves, namely that the leaves of one precede all leaves of the other, as this is the only way that high level operations can precede each other.
Example: Level-to-level Schedules

has L₂-to-L₁ and L₁-to-L₀ schedules:
Example: Non-reducible Layered Schedule with CSR Level-to-level Schedules

This shows that CSR is not a sufficient condition for reducibility.

The problem is that the equivalent serial order changes the ordering between conflicting and ordered \( f_{12}(y) \) and \( g_{22}(y) \). This is not allowed by def. of tree reducibility.

with \( f \) and \( g \) in conflict on same object, and \( h \) commuting with \( f \), \( g \), and \( h \).
Proof Sketch for Theorem 6.2

Consider adjacent levels $L_i, L_{(i-1)}$:
- CSR of the $L_i$-to-$L_{(i-1)}$ schedules allows isolating the $L_i$ ops
- Conflicting $L_i$ ops $f, g$ are not reordered:
  - Because of the $L_i$ conflict and the $L_{(i+1)}$-to-$L_i$ schedule being CSR, $f$ and $g$ must be ordered
  - Because of the $L_i$-to-$L_{(i-1)}$ schedule being OCSR this order is not reversed by the $L_i$-to-$L_{(i-1)}$ serialization
Example: Reducible Layered Schedule with Non-OCSR Level-to-level Schedules

with \( f \) and \( g \) in conflict, and \( h \) commuting with \( f, g, \) and \( h \)

This shows that OCSR is not a necessary condition

The last 3 L1 transactions are conflict serializable as: \( h22, h31, h12 \) not OCSR. Still tree-reducible as \( h \)'s commute.
Sufficient Conditions for Tree Reducibility

**Definition 6.13 (Conflict Faithfulness):**
A layered schedule $s = (\text{op}(s), <_s)$ is **conflict-faithful** if for each pair $p, q \in \text{op}(s)$ s.t. $p, q$ are non-commutative and for each $i > 0$ there is at least one operation pair $p', q'$ s.t. $p'$ and $q'$ are descendants of $p$ and $q$ with distance $i$ and are in conflict.

**Theorem 6.3:**
Let $s$ be an $n$-level schedule. If $s$ is conflict-faithful and for each $i, 0 < i \leq n$, the $L_i$-to-$L_{(i-1)}$ schedule derived from $s$ is in CSR, then $s$ is tree-reducible.
Proof Sketch for Theorem 6.3

Consider adjacent levels $L_i, L_{(i-1)}$:

- CSR of the $L_i$-to-$L_{(i-1)}$ schedules allows isolating the $L_i$ ops
- Conflicting $L_i$ ops $f, g$ are not reordered:
  - Because of the $L_i$ conflict and the $L_{(i+1)}$-to-$L_i$ schedule being CSR, $f$ and $g$ must be ordered, say $f < g$
  - Because of conflict-faithfulness $f$ and $g$ must have conflicting children $f', g'$ with $f' < g'$
  - CSR cannot reverse the order of $f'$ and $g'$, so the $L_i$-to-$L_{(i-1)}$ serialization must be compatible with the $L_i$ order $f < g$
Example: Reducible Layered Schedule

with Non-OCSR Level-to-level Schedules. Conflict Faithful.

with $f$ and $g$ in conflict,
and $h$ commuting with $f$, $g$, and $h$

This shows that OCSR is not a necessary condition

The last 3 L1 transactions are conflict serializable as: $h_2, h_3, h_1$ not OCSR. Still tree-reducible as $h$’s commute.
Example: Reducible Layered Schedule with Conflicting, Concurrent Operations

The argument: the low level execution is conflict equivalent to \( \text{fetch}_{21}(x) \ \text{modify}_{11}(x) \). So this may be assumed as the actual order (nothing in the system contradicts it). But then we can push \( \text{modify}_{11}(x) \) to the right and obtain \( t_2 < t_1 \).
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Definition 6.14 (State-Dependent Commutativity):
Operations p and q on the same object are **commutative in object state** \( \sigma \) if for all operation sequences \( \omega \)
the return parameters in the sequence \( pq\omega \) applied to \( \sigma \)
are identical to those in \( qp\omega \) applied to \( \sigma \).

**Example:**
- \( \sigma \): \( x\).balance = 40
  - \( s \): withdraw\(_1\)(x, 30) deposit\(_2\)(x,50) deposit\(_2\)(y,50) withdraw\(_1\)(y,30)
    \( \rightarrow \) would allow commuting the first step with both steps of \( t_2 \)
- \( \sigma \): \( x\).balance = 20
  - \( s \): withdraw\(_1\)(x, 30) deposit\(_2\)(x,50) deposit\(_2\)(y,50) withdraw\(_1\)(y,30)
    \( \rightarrow \) would not allow commuting the first two steps
Return-value Commutativity

Definition 6.18 (Return Value Commutativity):
An operation execution $p (\downarrow x_1, ..., \downarrow x_m, \uparrow y_1, ..., \uparrow y_n)$ is **return-value commutative** with an immediately following operation execution $q (\downarrow x'_1, ..., \downarrow x'_m, \uparrow y'_1, ..., \uparrow y'_n)$ if for every possible sequences $\alpha$ and $\omega$ s.t. $p$ and $q$ have indeed yielded the given return values in $\alpha pq\omega$, all operations in the sequence $\alpha qp\omega$ yield identical return values.

**Example:**
- $\sigma$: $x$.balance = 40
  $s$: withdraw$_1$(x, 30)$\uparrow$ok deposit$_2$(x,50)$\uparrow$ok ...  
    $\rightarrow$ withdraw$\uparrow$ok is return-value commutative with deposit
- $\sigma$: $x$.balance = 20
  $s$: withdraw$_1$(x, 30)$\uparrow$no deposit$_2$(x,50) $\uparrow$ok ...  
    $\rightarrow$ withdraw$\uparrow$no is not return-value commutative with deposit
### Examples: Return-value Commutativity Tables

#### Bank Accounts (Counters):

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( (x,\Delta_2)^{\uparrow} \text{ok} )</th>
<th>( (x,\Delta_2)^{\uparrow} \text{no} )</th>
<th>( (x,\Delta_2)^{\uparrow} \text{ok} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>withdraw ( (x,\Delta_1)^{\uparrow} \text{ok} )</td>
<td>+</td>
<td>−</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>withdraw ( (x,\Delta_1)^{\uparrow} \text{no} )</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>deposit ( (x,\Delta_1)^{\uparrow} \text{ok} )</td>
<td>−</td>
<td>+</td>
<td>+</td>
<td></td>
</tr>
</tbody>
</table>

#### Queues:

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( \text{enq}^{\uparrow} \text{ok} )</th>
<th>( \text{enq}^{\uparrow} \text{one} )</th>
<th>( \text{deq}^{\uparrow} \text{ok} )</th>
<th>( \text{deq}^{\uparrow} \text{empty} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{enq}^{\uparrow} \text{ok} )</td>
<td>−</td>
<td>impossible</td>
<td>+</td>
<td>impossible</td>
<td></td>
</tr>
<tr>
<td>( \text{enq}^{\uparrow} \text{one} )</td>
<td>−</td>
<td>impossible</td>
<td>−</td>
<td>impossible</td>
<td></td>
</tr>
<tr>
<td>( \text{deq}^{\uparrow} \text{ok} )</td>
<td>+</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td></td>
</tr>
<tr>
<td>( \text{deq}^{\uparrow} \text{empty} )</td>
<td>−</td>
<td>−</td>
<td>impossible</td>
<td>+</td>
<td></td>
</tr>
</tbody>
</table>
Example: Schedule on Counter Objects

$\text{t}_1 \quad \text{t}_2$

\begin{align*}
\text{decr(x,20)} & \uparrow \text{no} \\
\text{r(p)} & \\
\text{x=15} & \\
\text{y=45} & \\
\text{incr(x,30)} & \uparrow \text{ok} \\
\text{r(p)} & \\
\text{w(p)} & \\
\text{x=15} & \\
\text{y=45} & \\
\text{decr(y,20)} & \uparrow \text{ok} \\
\text{r(p)} & \\
\text{w(p)} & \\
\text{x=45} & \\
\text{y=45} & \\
\text{incr(y,30)} & \uparrow \text{no} \\
\text{r(p)} & \\
\text{x=15} & \\
\text{y=25} & \\
\end{align*}

Equivalent to serial order $\text{t}_1 < \text{t}_2$

with constraints $0 \leq x \leq 50$, $0 \leq y \leq 50$
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Lessons Learned

• Commutativity and abstraction arguments lead to the fundamental criterion of tree reducibility
• For layered schedules, CSR can be iterated from level to level
• Compared to page-model CSR, concurrency can be improved, potentially by orders of magnitude
• State-based commutativity can further enhance concurrency, but is more complex to manage