Assignment 2
Due December 10, 2018

Question 1: Hypergraph Acyclicity

In this question, you will devise an algorithm for constructing a join tree, if one exists, of a given hypergraph.

Question 1.1

Prove the following about a hypergraph $\mathcal{H}$ with a nonempty set of hyperedges.

1. If $\mathcal{H}$ has a join tree, then $\mathcal{H}$ has an ear.
2. If $\mathcal{H}$ has a join tree and we remove an ear from $\mathcal{H}$, then the resulting hypergraph has a join tree, or it has no hyperedges at all.

Question 1.2

Use the insights from the previous question to devise an efficient algorithm for constructing a join tree, if one exists, of a given hypergraph.

Question 2: Tree Decomposition

Let $G$ be a graph, and let $(T, \chi)$ be a tree decomposition of $G$.

1. Let $C$ be clique of $G$. Prove that $(T, \chi)$ contains a bag $\chi(t)$ that contains $C$ (i.e., $C \subseteq \chi(t)$).
2. Conclude that the treewidth of a graph is at least the maximum clique size, minus one.
3. Show an example of a graph where the treewidth is strictly greater than the maximum clique size minus one. Explain the correctness of your example.

Hint for 1. Assume that $T$ is directed top-down (with the root at the top), and consider the deepest node $t$ that contains $C$ underneath. What can be said about $t$?

Question 3: Loomis-Whitney Joins

Consider the Loomis-Whitney join $Q^k_{\text{LW}}$, as defined in class.

1. What is the generalized hypertree width of $Q^k_{\text{LW}}$?
2. What is the asymptotic maximal size of $Q^k_{\text{LW}}(D)$ for databases $D$ with $N$ tuples in each relation (as a function of $N$)? Prove your answer using the AGM bound.
Question 4: Yannakakis’s Algorithm

Consider the algorithm of Yannakakis for acyclic joins, as thought in class. Suppose that one simplifies the algorithm by skipping the first phase (inside-out full reduction), and applies only the second step (leaf-to-leaf join).

1. Would the simplified algorithm be correct?
2. Would the simplified algorithm be efficient?

Explain your answers.

Good luck!