Principles of Managing Uncertain Data

Lecture 10: Query Answering in Probabilistic Databases
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What is the outcome of “evaluating a query” over a probabilistic database?

Two semantics have been studied:

- Representation of the answer space
- Answer confidence (marginal probability)
Let \( \mathcal{P} \) be a probabilistic database over a signature \( \mathcal{R} \), and let \( Q \) be a query over \( \mathcal{R} \)

- Recall that a query \( Q \) maps every instance \( I \) over \( \mathcal{R} \) into a relation over the heading of \( Q \) (which is also a database)

- Then \( \mathcal{P} \) and \( Q \) define a new probabilistic database \( (\Omega, p) \), which we denote by \( \text{Eval}^{\text{rep}}(Q, \mathcal{P}) \):
  - \( \Omega = \{ Q(J) \mid J \text{ is a sample of } \mathcal{P} \} \)
  - \( p(J) = \Pr(Q(\mathcal{P}) = J) \)

Note the abuse of notation: inside \( \Pr(\cdot) \), we view \( \mathcal{P} \) as the random element that is equal to the random sample; hence, the meaning \( Q(\mathcal{P}) \) is the ordinary (deterministic) one
Closure of a Representation System

- Given a representation system REP and query language $QL$, we ask whether we can represent $\text{Eval}^{\text{rep}}(Q, P)$ in REP for every $Q \in QL$ and $P \in \text{REP}$.
- In this case we say that the representation system REP is \textit{closed} under the query language $QL$.
- A representation system is \textit{finitely complete} if it can represent every finite probabilistic database.
Recalling pc-Instances (by Example)

| ObsLects |  | ObsTAs |  | Courses |
|----------|  |--------|  |---------|
| lecturer |  | ta     |  | course  |
| Ahuva    | $x$ | Asma  | $x$ | DB      |
| Avia     | $\neg x \lor y$ | Alon  | $\neg x \lor z$ | PL |
|          |  |        |  | Ahuva   |
|          |  |        |  | Alon    |
|          |  |        |  | Avia    |
|          |  |        |  | Asma    |

$\pi(x, \text{true}) = 0.5$  $\pi(y, \text{true}) = 0.3$  $\pi(z, \text{true}) = 0.4$

$\pi(x, \text{false}) = 0.5$  $\pi(y, \text{false}) = 0.7$  $\pi(z, \text{false}) = 0.6$
Completeness of pc-Instances

**Theorem**

The class of pc-Instances (with Boolean variables) is a finitely complete representation system.

**Corollary**

The class of pc-Instances is closed under the relational algebra.
In the semantics of *answer confidence*, we associate each answer (tuple) with its individual (marginal) probability:

\[
\text{EVAL}^{\text{mrg}}(Q, \mathcal{P}) \overset{\text{def}}{=} \{(a, p) \mid a \in Q(J) \text{ for some sample } J \text{ and } p = \Pr(a \in Q(\mathcal{P}))\}
\]

That is, we concatenate to each tuple the probability that it is indeed an answer in a random possible world.
Example

<table>
<thead>
<tr>
<th>ObsLects</th>
<th>ObsTAs</th>
<th>Courses</th>
</tr>
</thead>
<tbody>
<tr>
<td>lecturer</td>
<td>ta</td>
<td>course</td>
</tr>
<tr>
<td>Ahuva</td>
<td>Asma</td>
<td>DB, PL</td>
</tr>
<tr>
<td>0.5</td>
<td>0.4</td>
<td>Ahuva, Alon</td>
</tr>
<tr>
<td>Avia</td>
<td>Alon</td>
<td>Avia, Asma</td>
</tr>
<tr>
<td>0.3</td>
<td>0.9</td>
<td></td>
</tr>
</tbody>
</table>

Which are the courses with their full staff observed?

\[ Q(z) :\neg \text{ObsLects}(x), \text{ObsTAs}(y), \text{Courses}(z, x, y) \]

DB: 0.5 \times 0.9 \quad PL: 0.3 \times 0.4

Does any course have a full staff observed?

\[ Q() :\neg \text{ObsLects}(x), \text{ObsTAs}(y), \text{Courses}(z, x, y) \]

Yes: \[ 1 - (1 - 0.5 \times 0.9) \times (1 - 0.3 \times 0.4) \] \quad No: 1 - Yes
We will focus on the semantics $EVAL^{mrg}(Q, \mathcal{P})$

Various reasons:

- Representation systems with interesting closure properties are capable of expressing significant correlations, and hence, cast query evaluation highly intractable
- We will see that query evaluation is challenging already over tuple-independent databases, which do not have any interesting closure properties
- There is a rich literature on $EVAL^{mrg}(Q, \mathcal{P})$
  - Not so much on $EVAL^{rep}(Q, \mathcal{P})$
We Need Appropriate Complexity Machinery

- Query evaluation in probabilistic databases entails computing/estimating probabilities (a.k.a. probabilistic inference)
- To understand and analyze the complexity of numerical computations, we need appropriate complexity measures
  - The standard theory from undergrad studies is mainly about deciding yes/no, not about computing numbers
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Types of Computational Problems

- A decision problem is a mapping $F : \{0, 1\}^* \rightarrow \{0, 1\}$
  - For example, is the given string representing a:
    - connected graph?
    - satisfiable CNF formula? (CNF-SAT)
    - database that satisfies some fixed Boolean query?

- A function problem is a mapping $F : \{0, 1\}^* \rightarrow \{0, 1\}^*$
  - For example:
    - Mapping a (representation of a) given graph to (a representation of) its connected components
    - Mapping a given CNF formula into the number of its satisfying truth assignments
    - Mapping a tuple-independent probabilisitic DB into the probability that it satisfies some fixed Boolean query

- The natural associated task: given $x \in \{0, 1\}^*$, compute $F(x)$
A **complexity class** is a class $\mathcal{C}$ of computational problems such that each problem in $\mathcal{C}$ can be solved by a Turing machine that satisfies some *properties*. These properties are typically limitations on the machine’s power, e.g., determinism, bound on the execution time or used space.
Examples of Complexity Classes

- **P**: Decision problems solvable in polynomial time
  - That is, problems \( D \) where there exists a Turing machine \( M \) and a polynomial \( p \), such that \( M \) solves \( D \) and \( M \) terminates on input \( x \) after at most \( p(|x|) \) steps

- **FP**: Function problems solvable in polynomial time

- **NP**: Decision problems solvable in polynomial-time by a *non-deterministic* Turing machine
  - That is, \( B(x) = 1 \) if and only if there is an *accepting path* on \( x \)

- **coNP**: Decision problems with complement in NP
NP Relations

- An **NP relation** is a relation \( R \subseteq \{0, 1\}^* \times \{0, 1\}^* \) that satisfies the following:
  - Membership (of a given pair) in \( R \) can be decided in polynomial time
  - There is a polynomial \( p \) such that for every \((x, y) \in R\) we have \(|y| \leq p(|x|)\)
- If \((x, y) \in R\), then we often refer to \( y \) as a **witness** for \( x \)
- **NP** can be described as the class of all decision problems of the form
  \[
  B(x) = \begin{cases} 
  1 & \text{if } \exists y[(x, y) \in R]; \\
  0 & \text{otherwise.}
  \end{cases}
  \]
  where \( R \) is an NP relation
Leslie Valiant (2010 Turing Award Winner) has established complexity theory for *counting problems* [Val79]

- $\#P$ is the class of problems that count the number of witnesses in an NP relation; formally, $\#P$ contains all function problems of the following form, where $R$ is an NP relation:

$$F(x) = |\{y \mid (x, y) \in R\}|$$

- Equivalently: count the number of accepting paths of a polynomial-time non-deterministic Turing machine.
Reductions

- Let $H$ and $W$ be two computational problems
  - Typically, $H$ is some hard problem, and $W$ is a problem that we wish to prove hardness for
- A reduction of $H$ to $W$ is a way of showing:
  - We could solve $H$ if we could solve $W$
  - $W$ is “at least as hard” as $H$
- There is more than one way to define a reduction
Types of Reductions

- The common type of (polynomial-time) reduction from $H$ to $W$ is that of *Karp reduction*:

  Given input for $H$, transform it (in polynomial time) into input for $W$, such that both inputs have the same answer.

- Another common type (more applicable to function problems) is a polynomial time *Turing reduction* (a.k.a. *Cook reduction*):

  Given input $x$ for $H$, solve it in polynomial time using $W$ via an oracle: an $O(1)$ subroutine that returns $W(x')$ given $x'$.
A function problem $F$ is **hard for $\#P$** (or $\#P$-hard) if for every problem $G$ in $\#P$ there is a *Turing reduction* from $G$ to $F$.

If $F$ itself is in $\#P$, then $F$ is **$\#P$-complete**.
Examples of \#P-Complete Problems

- \#CNF: Number of satisfying assignments to a given CNF
- \#DNF: Number of \ldots\text{DNF} \quad \text{Why?}
- Number of tuples in \(Q(I)\), when \(Q\) is a given CQ and \(I\) is a given database instance
- Number of vertex covers in a graph
- Number of perfect matchings in a bipartite graph (a.k.a. the permanent of a 0/1 matrix)
- Number of paths between two given nodes in a graph
- Number of topological orderings for a given DAG
How Hard is \#P-Hard?

- What can we do with an oracle to a \#P-hard problem?
- More formally, which problems are Turing-reducible to, e.g., \#DNF?
  - Every problem in NP, and every problem in coNP
  - In fact, every problem in the *polynomial hierarchy* (Toda’s Theorem [TO92]), for example:

  \[
  \text{Given a CNF formula } \varphi(x, y, z), \text{ is there a tuple } a \text{ such that for every tuple } b \text{ there exists a tuple } c \text{ such that } \varphi(a, b, c)\
  \]
Counting via Statistical Inference

- How is probability related to counting?
- We explain by an example
- Consider a propositional formula $\varphi(x_1, \ldots, x_n)$
- Now suppose that every variable takes a random truth value:
  $\Pr(x_i = \text{true}) = \Pr(x_i = \text{false}) = 0.5$
- Also assume that the $x_i$’s are probabilistically independent
- Could you compute $\Pr(\varphi(x))$?
A Turing Reduction

\[ \varphi(x_1, \ldots, x_n): \text{reduce counting to probability computation} \]

\[
\begin{align*}
\Pr(\varphi(x)) & = \sum_{a \mid \varphi(a)} \Pr(x = a) = \sum_{a \mid \varphi(a)} 0.5^n = 2^{-n} \cdot |\{a \mid \varphi(a)\}| \\
\end{align*}
\]

We get the following Turing reduction:

To compute the number of satisfying assignments for \( \varphi \), compute \( \Pr(\varphi(x)) \) and multiply by \( 2^n \)

Note: \( 2^n \) is represented by \( n \) bits; multiplication in PTime in \( n \)

Hence, computing the probability of a given propositional formula over given random Boolean variables is hard for \( \#P \)
Comment on Numerical Representation

- For complexity analysis, we need to assume something about the representation of numbers
- We will assume that numbers are given in their *rational* representation, namely, $a = n/m$ is represented as $(n, m)$
  - Each of $n$ and $m$ is given in binary

Numerical representation will not be an issue for us; but generally speaking, ignoring the size of numerical representation may easily lead to wrong statements
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Let $Q$ be a Boolean CQ without self joins.

For a variable $x \in \text{Var}(Q)$, denote by $\text{Atoms}(x)$ the set of all $\alpha \in \text{Atoms}(Q)$ such that $x$ occurs in $\alpha$; in notation:

$$\text{Atoms}(x) \overset{\text{def}}{=} \{ \alpha \in \text{Atoms}(Q) \mid x \in \text{Var}(\alpha) \}$$

We say that $Q$ is hierarchical if for every two variables $x$ and $y$ in $Q$, at least one of the following holds:

- $\text{Atoms}(x) \subseteq \text{Atoms}(y)$
- $\text{Atoms}(y) \subseteq \text{Atoms}(x)$
- $\text{Atoms}(x) \cap \text{Atoms}(y) = \emptyset$
Examples

- Every CQ with two or fewer atoms is hierarchical
- $R(x), S(x, y), T(y)$ is not hierarchical; Why?
- The following is hierarchical:

$$R(x, y, u), S(x, y, z), T(x), U(w), V(w, v)$$

- Fact: If $Q$ is hierarchical, then every subquery of $Q$ (subset of $Q$'s atoms) is hierarchical
Dichotomy for Boolean CQs Without Self Joins

Dichotomy [DS04]

Let $Q$ be a Boolean CQ without self joins.

- If $Q$ is hierarchical, then it can be evaluated in polynomial time over tuple-independent databases.
- Otherwise, its evaluation over tuple-independent databases is \#P-hard.
Proof of Hardness Side

Two parts:

1. Prove \#P-hardness for a specific CQ:

   \[ Q_{RST}() \equiv R(x), S(x, y), T(y) \]

2. Prove \#P-hardness for an arbitrary CQ \( Q \) w/o self joins that is non-hierarchical . . . by reducing \( Q_{RST} \) evaluation to \( Q \) evaluation
Lemma

Evaluating $Q_{RST}$ over a tuple-independent database is \#P-hard.

Proof: Reduction from monotone bipartite \#DNF that has been proved hard [PB83]
A *monotone bipartite DNF* formula is a 2-DNF formula such that there are two *disjoint* sets $X$ and $Y$ of variables where each clause has the form $x \land y$ for $x \in X$ and $y \in Y$

Example: $(x_1 \land y_1) \lor (x_1 \land y_2) \lor (x_2 \land y_1) \lor (x_2 \land y_3)$

In particular, no negation

Monotone bipartite #DNF is the problem of counting how many satisfying assignments does a given such formula have

Equivalently: *given a bipartite graph, how many subsets of the nodes include at least one edge?*
### Reduction

\[
Q_{RST}(Q) \leftarrow R(x), S(x, y), T(y)
\]

\[
(x_1 \land y_1) \lor (x_1 \land y_2) \lor (x_2 \land y_1) \lor (x_2 \land y_3)
\]

<table>
<thead>
<tr>
<th>R</th>
<th>A</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x_1</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>x_2</td>
<td>0.5</td>
</tr>
</tbody>
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<table>
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<tr>
<th>S</th>
<th>A</th>
<th>B</th>
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<tr>
<td></td>
<td>x_1</td>
<td>y_1</td>
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<td>x_1</td>
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<td>x_2</td>
<td>y_1</td>
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<td>x_2</td>
<td>y_3</td>
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<thead>
<tr>
<th>T</th>
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<tr>
<td></td>
<td>y_1</td>
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<tr>
<td></td>
<td>y_2</td>
</tr>
<tr>
<td></td>
<td>y_3</td>
</tr>
</tbody>
</table>

- Suppose that there are \( n \) tuples in \( R \) and \( T \) combined
- What is the probability of every possible world?
- What is the probability of \( Q_{RST} \)?
Part 2

- Let $Q$ be an arbitrary CQ such that:
  - $Q$ has no self joins
  - $Q$ is non-hierarchical
- Claim: we can choose variables $x$ and $y$ and atoms $\alpha_x$, $\alpha_y$ and $\alpha_{x,y}$ such that:
  - $x \in \text{Var}(\alpha_x)$ and $y \notin \text{Var}(\alpha_x)$
  - $y \in \text{Var}(\alpha_y)$ and $x \notin \text{Var}(\alpha_y)$
  - $x, y \in \text{Var}(\alpha_{x,y})$
- *(Discussion until everyone is convinced)*
- W.l.o.g. we assume that in both $Q_{RST}$ and $Q$ these are the same variable names $x$ and $y$
- We will show how to reduce query evaluation over $Q_{RST}$ to query evaluation over $Q$
Example of Reduction

\[ R(x), S(x, y), T(y) \]

\[ \downarrow \]

\[ U(x, z), V(x, u), W(x, y, z), Y(y, a) \]

\[ \alpha_x \]
\[ \alpha_{xy} \]
\[ \alpha_y \]
### Example of Reduction

Let's consider the following relations:

- **R(x)**, **S(x, y)**, **T(y)**

**R**

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>(p_1)</th>
<th>(p_2)</th>
</tr>
</thead>
</table>

**S**

| 1 | 2 | 3 | \(q_{13}\) | \(q_{23}\) |

**T**

| 3 | 4 | \(r_3\) | \(r_4\) |


\[ \Downarrow \]

**U**

| 1 | 2 | c | \(p_1\) | \(p_2\) |

**V**

| 1 | 2 | c | 1 |

**W**

| 1 | 2 | 3 | 4 | c | \(q_{13}\) | \(q_{23}\) |

**Y**

| 3 | 4 | a | \(r_3\) | \(r_4\) |

The resulting relations are:

- **U(x, z)**, **V(x, u)**, **W(x, y, z)**, **Y(y, a)**

Where:

- \(\alpha_x\)
- \(\alpha_{xy}\)
- \(\alpha_y\)
Reduction

- We are given a tuple-independent database \( \mathcal{P} = (I, \pi) \), and we wish to evaluate \( \Pr(\mathcal{P} \models Q_{rst}) \) by reducing to query evaluation over \( Q \).
- We will construct a tuple-independent database \( \mathcal{P}' = (I', \pi') \) such that

\[
\Pr(\mathcal{P} \models Q_{rst}) = \Pr(\mathcal{P}' \models Q)
\]
Constructing $I'$

- Start with an empty $I'$
- Select some constant value $c$
- For each homomorphism (or grounding) $\mu$ from $Q_{RST}$ to $I$ and atom $\alpha \in \text{Atoms}(Q)$
  1. Extend $\mu$ to a mapping over $\text{Var}(Q)$ by mapping every variable other than $x$ and $y$ to $c$
  2. Create the fact $f = \mu(\alpha)$
  3. Add $f$ into $I'$
Constructing $\pi'$

- Recall: $Q_{RST}() :- R(x), S(x, y), T(y)$
- For each extension of a homomorphism $\mu$ from $Q_{RST}$ to $I$ and atom $\alpha \in \text{Atoms}(Q)$ we define:

$$
\pi' (\mu(\alpha)) = \begin{cases} 
\pi(R(\mu(x))) & \text{if } \alpha = \alpha_x; \\
\pi(S(\mu(x), \mu(y))) & \text{if } \alpha = \alpha_{xy}; \\
\pi(T(\mu(y))) & \text{if } \alpha = \alpha_y; \\
1 & \text{otherwise.}
\end{cases}
$$
How do we prove that $\Pr(P \models Q_{RST}) = \Pr(P' \models Q)$?

First, we can ignore all the facts of $I$ that are not in the image of homomorphisms from $Q_{RST}$; with that:

There is a (straightforward) bijection $\varphi$ between the samples $J$ of $P$ and $P'$ such that:

- $\Pr(J) = \Pr(\varphi(J))$
- $J \models Q_{RST}$ if and only if $\varphi(J) \models Q$

(Discussion until everyone is convinced)

From that, the conclusion $\Pr(P \models Q_{RST}) = \Pr(P' \models Q)$ is simply due to the definition of these probabilities.
Next, we prove the tractability side:

If $Q$ is hierarchical, then it can be evaluated in polynomial time over tuple-independent databases.
Let $Q$ be a hierarchical CQ without self joins

- A **root variable** of $Q$ is a variable $x \in \text{Var}(Q)$ such that $\text{Atoms}(x)$ is maximal w.r.t. set containment.

**Which are the root variables in $R(x, y), S(x), T(z)$?**
Recursive algorithm

Input: tuple-independent $\mathcal{P} = (I, \pi)$

```plaintext
if $\text{Var}(Q) = \emptyset$ then
    return $\Pr(\mathcal{P} \models Q)$ // How?

Select a root variable $x$

if $\text{Atoms}(x) \not\subseteq \text{Atoms}(Q)$ then
    Apply an independent-join reduction
else
    Apply an independent-project reduction
```
Independent-Join Reduction

- Suppose that $x$ is a root variable that occurs in some (but not all) of the atoms
- Write $Q = Q_x \land Q'$ where $Q_x$ consists of the atoms that contain $x$, and $Q'$ consists of the remaining atoms
- Claim: $Q_x$ and $Q'$ do not share any atoms or variables
  - *(Discussion until everyone is convinced)*
- Therefore, the events $\mathcal{P} \models Q_x$ and $\mathcal{P} \models Q'$ are probabilistically independent
  - *(Discussion until everyone is convinced so far)*
- Hence, $\Pr(\mathcal{P} \models Q) = \Pr(\mathcal{P} \models Q_x) \times \Pr(\mathcal{P} \models Q')$
- We reduced from $Q$ to $Q_x$ and $Q'$, each having fewer atoms
Independent-Project Reduction

- Suppose that \( x \) is a variable that occurs in \textit{every} atom
- Let \( A \) be the set of all the constants that \( x \) can take in \( I \)
- We have the following:

\[
\Pr(\mathcal{P} \models Q) = 1 - \prod_{a \in A} \left( 1 - \Pr(\mathcal{P} \models Q_{[x \rightarrow a]}) \right)
\]

- \textit{(Discussion until everyone is convinced)}
- We reduced from \( Q \) to \( Q_{[x \rightarrow a]} \), which \textit{has fewer variables}
- Claim: \( Q_{[x \rightarrow a]} \) is hierarchical
Detailed Algorithm

Input: tuple-independent $\mathcal{P} = (I, \pi)$

\[
\text{Eval}(Q)(\mathcal{P})
\]

if $\text{Var}(Q) = \emptyset$ then
  return $\Pr(\mathcal{P} \models Q)$

Select a root variable $x$

if $\text{Atoms}(x) \not\subseteq \text{Atoms}(Q)$ then
  Return $\text{Eval}(Q_A)(\mathcal{P}) \times \text{Eval}(Q')(\mathcal{P})$

else
  Return $1 - \prod_{a \in A} (1 - \text{Eval}(Q_{[x \rightarrow a]})(\mathcal{P}))$

That’s it!
Discussion on Complexity

Why is the algorithm considered “polynomial time”?

What is the dependence on the size of the CQ?
Dichotomy for non-Boolean CQs Without Self Joins

Generalized Dichotomy [DS04]

Let $Q(x)$ be a CQ without self joins. Let $Q_b$ be the Boolean CQ $Q[x \rightarrow a]$ for some tuple $a$ of constants.

- If $Q_b$ is hierarchical, then $Q$ can be evaluated in polynomial time over tuple-independent database.
- Otherwise, evaluating $Q$ is $\#P$-hard.
Mark of the following as $\text{PTime} / \#\text{P-hard}$:

<table>
<thead>
<tr>
<th>Query</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q() :- R(x), S(x, y), T(y)$</td>
<td>$#\text{P-hard}$</td>
</tr>
<tr>
<td>$Q(y) :- R(x), S(x, y), T(y)$</td>
<td>$\text{PTime}$</td>
</tr>
<tr>
<td>$Q(x) :- R(x, z), S(x, y), T(y, z)$</td>
<td>$#\text{P-hard}$</td>
</tr>
<tr>
<td>$Q(y) :- R(x, z), S(x, y), T(y, z)$</td>
<td>$#\text{P-hard}$</td>
</tr>
<tr>
<td>$Q(x, y) :- R(x, z), S(x, y), T(y, z)$</td>
<td>$\text{PTime}$</td>
</tr>
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</table>
Suciu and Dalvi generalized their dichotomy into general UCQs (and in particular CQs with self joins) [DS12].

- The generalized dichotomy has:
  - A substantially more involved separation condition
  - A substantially longer (87 pages) and more complicated proof
Let $\mathcal{R}$ be a signature

Given: a CQ $Q(x)$ over $\mathcal{R}$ such that

- $Q$ has no self joins
- $Q_b$ is hierarchical
Let $\mathcal{R}$ be a signature

- Given: a CQ $Q(x)$ over $\mathcal{R}$ such that
  - $Q$ has no self joins
  - $Q_b$ is hierarchical

- Goal: Compute a SQL query $Q_p(x, z)$ over $\mathcal{R}^+$, such that for every tuple-independent database $\mathcal{P} = (I, \pi)$ over $\mathcal{R}$ we have:

$$Q_p(I^+) = \{(a, p) \mid a \in Q(I) \land p = \Pr(a \in Q(\mathcal{P}))\}$$

where $\mathcal{R}^+$ is obtained from $\mathcal{R}$ by adding a probability attribute to each relation symbol, and $I^+$ is obtained from $I$ by setting this attribute according to $\pi$. 
We denote $Q$ as the following SQL query:

```
SELECT X FROM R WHERE AC
```

Where:
- $R$ is a sequence $R_1, \ldots, R_m$ of relation names with a distinguished attribute $P$
- $X$ is a sequence of attributes $A$
  - For simplicity, we assume that different relations use disjoint sets of attribute names, except for $P$
- $AC$ is a conjunction of conditions of the form $A = B$ or $A = t$ where $t$ is a constant
- None of the above attributes $A$ and $B$ is the distinguished $P$
As a first step, we will extend $X$ with *every* attribute that corresponds to an output variable; call the result $X^+$

```
SELECT X,P FROM ViewFor(Q, X+)
```

We will apply the recursive algorithm by building multiple views of the form $\text{ViewFor}(Q_0, X_0)$
Base Case

CREATE VIEW ViewFor(Q, Y) AS
SELECT Y, R1.P * ... * Rm.P AS P
FROM R
WHERE AC

Recall: in this case all the attributes are in the output (except for those that are constants in the CQ)
Independent-Join Reduction

CREATE VIEW ViewFor(Q, Y) AS
SELECT Y, Q1.P * Q2.P AS P
FROM
    ViewFor(Q_x, Y ∩ Var(Q_x)) Q1,
    ViewFor(Q', Y ∩ Var(Q')) Q2
WHERE AC ∩ Y

AC ∩ Y denotes the restriction of AC to comparisons involving attributes only from Y
CREATE VIEW ViewFor\((Q, Y)\) AS
SELECT \(Y, 1-\exp(\text{SUM}(\ln(1-Q1.P)))\) AS \(P\)
FROM ViewFor\((Q, Y \cup Z_x)\) Q1
GROUP BY \(Y\)

- \(\exp(\text{SUM}(\ln(x)))\) means \(\text{Prod}(x)\), which is not supported by PSQL
- \(Z_x\) consists of all the attributes where \(x\) occurs
- Recall that \(x\) is a root variable that occurs in every atom
Example

\[ Q(z) \leftarrow R(x, y), S(x, z), T(z, a) \]

Representation with disjoint attribute sets:

\[ R(x_R, y_R, P) \quad S(x_S, z_S, P) \quad T(z_T, a_T, P) \]

\[
\text{SELECT } z_S \\
\text{FROM } R, S, T \\
\text{WHERE } x_R = x_S \text{ AND } z_S = z_T \text{ AND } a_T = 'a'
\]
Step 0

SELECT zS
FROM R, S, T
WHERE xR=xS AND zS=zT AND aT='a'

⇓

SELECT zS, P FROM ViewFor(Q,zS,zT)
CREATE VIEW ViewFor($Q$, zS, zT) AS
SELECT zS, zT, Q1.P * Q2.P AS P
FROM
    ViewFor($R(\langle x, y \rangle, S(\langle x, z \rangle))$, zS) Q1,
    ViewFor($T(z, a)$, zT) Q2
WHERE zS = zT
ViewFor((\(R(x, y), S(x, z)\)),zS) via Independent-Project Reduction

CREATE VIEW ViewFor((\(R(x, y), S(x, z)\)),zS) AS
SELECT zS, 1-EXP(SUM(LN(1-Q1.P))) AS P
FROM ViewFor((\(R(x, y), S(x, z)\)),zS,xR,xS) Q1
GROUP BY zS
ViewFor((R(x, y), S(x, z)), zS, xR, xS)) via Independent-Join Reduction

CREATE VIEW ViewFor((R(x, y), S(x, z)), zS, xR, xS) AS
SELECT zS, xR, xS, Q1.P * Q2.P AS P
FROM
    ViewFor(R(x, y), xR) Q1, ViewFor(S(x, z), zS, xS) Q2
WHERE xR = xS
Create view ViewFor($R(x,y), xR$) as independent-project reduction:

```sql
CREATE VIEW ViewFor($R(x,y), xR$) AS
    SELECT xR, 1-EXP(SUM(LN(1-Q1.P))) AS P
    FROM ViewFor($R(x,y), xR, yR$) Q1
    GROUP BY xR
```
ViewFor($R(x,y), xR, yR$) via Base Case

CREATE VIEW ViewFor($R(x,y), xR, yR$) AS
SELECT xR, yR, P
FROM R

- It simply copies $R$!
- Similar to ViewFor($S(x,z), zS, xS$)


CREATE VIEW ViewFor($T(z, a), zT$) AS
SELECT zT, P
FROM T
WHERE aT='a'

ViewFor($T(z, a), zT$) via Base Case
Table of Contents

1. Querying Probabilistic Databases
2. Complexity Basics
3. Dichotomy in CQ Complexity
4. Approximate Query Evaluation
5. Recap
Let $a$ and $b$ be two numbers, and let $\epsilon > 0$ be a small number (e.g., 0.01).

- We say that $b$ is an \textit{additive} (or \textit{absolute}) approximation of $a$ with error $\epsilon$ if

$$a - \epsilon < b < a + \epsilon$$

- We say that $b$ is a \textit{multiplicative} (or \textit{relative}) approximation of $a$ with error $\epsilon$ if

$$\frac{a}{1 + \epsilon} < b < (1 + \epsilon)a$$
Approximation Schemes

- Let $f : \{0, 1\}^* \to \mathbb{R}$ be a numeric function
- A *polynomial-time approximation scheme* (PTAS) for $f$ is an algorithm $A$ such that:
  - $A$ takes as input an input $x \in \{0, 1\}^*$ and $\epsilon > 0$, and computes an approximation of $f$ with error $\epsilon$
  - For every fixed $\epsilon > 0$, the running time of $A$ is polynomial in $|x|$
- A *fully polynomial-time approximation scheme* (FPTAS) for $f$ is a PTAS for $f$ that runs in time polynomial in $|x|$ and $1/\epsilon$
- Note: PTAS and FPTAS for $f$ mean that an approximation for $f$ is (theoretically) tractable for as small an error as one desires; in the case of FPTAS, the error has a “manageable” impact on the running time
Randomized Approximation Schemes

- Let $f : \{0, 1\}^* \to \mathbb{R}$ be a numeric function
- A *polynomial-time randomized approximation scheme* (PRAS) for $f$ is a randomized algorithm $A$ such that:
  - $A$ takes as input an input $x \in \{0, 1\}^*$ and $\epsilon > 0$, and with a probability of at least $2/3$ computes an approximation of $f$ with error $\epsilon$
  - For every fixed $\epsilon > 0$, the running time of $A$ is polynomial in $|x|$
- A *fully polynomial-time randomized approximation scheme* (FPRAS) for $f$ is a PRAS for $f$ that runs in time polynomial in $|x|$ and $1/\epsilon$
The reliability factor $2/3$ is arbitrary; every number greater than $1/2$ would work.

Given an FPRAS, we can decrease the probability of failure to any desired number (e.g., 0.001) by repeatedly applying the algorithm and taking the median number.

- By applying known concentration bounds (e.g., Hoeffding’s inequality), we get that the failure probability decreases exponentially in the number of trials.

We will prove it in a few slides.
Hoeffding’s Inequality

Consider \( n \) independent tosses of a coin with probability \( p \) of heads. Let \( H \) be the number of heads. For all \( \epsilon > 0 \) we have:

\[
\Pr \left( p - \epsilon < \frac{H}{n} < p + \epsilon \right) \geq 1 - 2e^{-2en}
\]

In particular, by running \( O(1/\epsilon) \) trials we get an additive approximation of \( p \) with probability greater than \( 2/3 \)
FPRAS Amplification (1)

- Let $f$ be a numeric function, and suppose that $A$ is an FPRAS (additive or relative) that gives an $\epsilon$-approximation with failure probability $\delta_A \leq 1/3$
- Also, suppose that we are given a number $\delta > 0$ (e.g., $\delta = 0.01$), and we wish to reduce failure probability to $\delta$
- Given input $x$, let $a_1, \ldots, a_n$ be random numbers that we get by $n$ independent runs of $A$
  - We decide later on what $n$ should be and how it depends on $\delta$
- Let $b_1, \ldots, b_n$ be $n$ Boolean random variables such that
  \[ b_i = \begin{cases} 
  1 & \text{if } a_i \text{ is an } \epsilon\text{-approximation of } f(x); \\
  0 & \text{otherwise.} 
\end{cases} \]
- $b_i$ are viewed as the tosses of a coin with probability $(1 - \delta_A) \geq 2/3$ for 1
Let $a$ be the median of $a_1, \ldots, a_n$

What is the probability that $a$ is *not* an $\epsilon$-approximation?

If $a$ is not an $\epsilon$-approximation, then at least half of the $a_i$ are not $\epsilon$-approximations, and at least half of the $b_i$'s are zero.

As usual, let $H$ be the number of 1s (heads); by Hoeffding’s:

$$\Pr(a \text{ not an } \epsilon\text{-approx}) \leq \Pr\left(\frac{H}{n} \leq \frac{1}{2}\right) \leq \Pr\left(\frac{H}{n} \leq (1 - \delta_A) - \frac{1}{6}\right)$$

$$\leq 2e^{-2\times(1/6)\times n} = 2e^{-n/3} < e^{-n/3+1}$$

To get the failure probability at most $\delta$, we need $e^{-n/3+1} \leq \delta$, or equivalently, $n \geq 3(\log \frac{1}{\delta} + 1)$
Consider a Boolean query $Q$, such that $Q$ can be evaluated in polynomial time.

Then there is an additive FPRAS for $Q$ over a pc-instance:

\[
\begin{align*}
c &= 0 \\
\text{for } i = 1, \ldots, n \text{ do} & \\
\quad \text{Sample a random possible world } J \\
\quad \text{if } Q(J) \text{ then} & \\
\quad \quad \quad + + c \\
\text{Return } c/n
\end{align*}
\]
We will show that there is a multiplicative FPRAS for every CQ (in fact, UCQ) over tuple-independent (in fact, BID) databases.

We will first translate the evaluation of a Boolean UCQ into the computation of a probability of a disjunctive event.

Then, we will show an FPRAS for disjunctive events, under assumptions that hold in our case.
Translating a Boolean UCQ into a Disjunctive Event

- Let $\mathcal{P}$ be a probabilistic database, where all the samples are subinstances of some instance $I$
- Let $Q$ be a UCQ
- The event “$\mathcal{P} \models Q$” can be phrased as a disjunction $E_1 \lor \cdots \lor E_m$ of events
  - Each $E_i$ corresponds to a set $F_i$ of facts in $I$, and states that “every fact in $F_i$ is in the sample”
  - The facts in $F_i$ are those obtained by applying a homomorphism from one of the CQs of $Q$ to $I$
    - In common terminology, each $E_i$ is a grounding of one of $Q$’s CQs
  - The number $m$ of these $E_i$ is polynomial in $|I|$, since we assume that $Q$ is fixed
### Example on a BID Database

<table>
<thead>
<tr>
<th>key</th>
<th>lecturer</th>
<th>student</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Asma</td>
<td>Ahuva</td>
<td>0.5</td>
</tr>
<tr>
<td>a</td>
<td>Alon</td>
<td>Avia</td>
<td>0.3</td>
</tr>
<tr>
<td>b</td>
<td>Asma</td>
<td>Anna</td>
<td>0.3</td>
</tr>
<tr>
<td>b</td>
<td>Adi</td>
<td>Ariel</td>
<td>0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>key</th>
<th>teacher</th>
<th>course</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Asma</td>
<td>DB</td>
<td>0.4</td>
</tr>
<tr>
<td>a</td>
<td>Asma</td>
<td>PL</td>
<td>0.3</td>
</tr>
<tr>
<td>b</td>
<td>Ahuva</td>
<td>OS</td>
<td>0.3</td>
</tr>
</tbody>
</table>

\[ Q() \leftarrow (\text{Advises}(u, x, y) \land \text{Teaches}(v, x, z)) \lor (\text{Advises}(u, y, x) \land \text{Teaches}(v, x, z)) \]

\( Q \) is equivalent to:

- \( E_1 \): both Advises(a,Asma,Ahuva) and Teaches(a,Asma,DB) are chosen; or
- \( E_2 \): both Advises(a,Asma,Ahuva) and Teaches(b,Ahuva,OS) are chosen; or
- \( E_3 \): both Advises(b,Asma,Anna) and Teaches(a,Asma,PL) are chosen; or
- \( \ldots \)
Karp-Luby-Madras Approximation for Disjunctions

- Let $\mathcal{P} = (\Omega, p)$ be a probability space and $E_1, \ldots, E_m$ a sequence of events in $\mathcal{P}$
  - All given in some representation
- Assume that for each $E_i$ we can efficiently:
  1. Compute the probability of $E_i$
  2. Test whether $E_i$ is true in a given random sample
  3. Sample from the subspace conditioned on $E_i$
- Claim: These assumptions hold in our case (UCQs over BIDs)
  - (Discussion until everyone is convinced)
- We will show the Karp-Luby-Madras FPRAS for $\Pr(\bigvee_{i=1}^m E_i)$ under these assumptions [KLM89]
\[
\Pr \left( \bigvee_{i=1}^{m} E_i \right) \overset{\text{easy}}{=} \Pr \left( \bigvee_{i=1}^{m} \neg E_1 \land \cdots \land \neg E_{i-1} \land E_i \right)
\]

\[
\overset{\text{disjoint}}{=} \sum_{i=1}^{m} \Pr \left( \neg E_1 \land \cdots \land \neg E_{i-1} \land E_i \right)
\]

\[
\overset{\text{definition}}{=} \sum_{i=1}^{m} \Pr(E_i) \times \Pr \left( \neg E_1 \land \cdots \land \neg E_{i-1} \mid E_i \right)
\]
Algorithm (2)

\[
\Pr \left( \bigvee_{i=1}^{m} E_i \right) = \sum_{i=1}^{m} \Pr(E_i) \times \Pr(\neg E_1 \land \cdots \land \neg E_{i-1} \mid E_i)
\]

- We can compute \(\Pr(E_i)\) efficiently
- We can additively approximate \(\Pr(\neg E_1 \land \cdots \land \neg E_{i-1} \mid E_i)\)
  - Using Hoeffding’s inequality, since we can sample the conditional subspace of each \(E_j\), and test for the conjunction in the sample
- Let \(a_i\) be an additive approx. of \(\Pr(\neg E_1 \land \cdots \land \neg E_{i-1} \mid E_i)\) with error \(\epsilon'\) and failure probability \(\delta\)
  - We will determine \(\epsilon'\) in the end
  - \(\delta\) should be small enough so that with probability at least \(2/3\) all the \(a_i\) are good
Algorithm (3)

We will approximate \( \Pr(\bigvee_{i=1}^{m} E_i) \) using \( \sum_{i=1}^{m} \Pr(E_i) \times a_i \)

\[
\sum_{i=1}^{m} \Pr(E_i) \times a_i \leq \sum_{i=1}^{m} \Pr(E_i) \times (\Pr(\neg E_1 \land \cdots \land \neg E_{i-1} \mid E_i) + \epsilon')
\]

\[
= \sum_{i=1}^{m} \Pr(E_i) \times \Pr(\neg E_1 \land \cdots \land \neg E_{i-1} \mid E_i) + \sum_{i=1}^{m} \Pr(E_i) \times \epsilon'
\]

\[
= \Pr\left(\bigvee_{i=1}^{m} E_i\right) + \epsilon' \sum_{i=1}^{m} \Pr(E_i) \leq \Pr\left(\bigvee_{i=1}^{m} E_i\right) + \epsilon' m \times \Pr\left(\bigvee_{i=1}^{m} E_i\right)
\]

\[
= (1 + \epsilon' m) \times \Pr\left(\bigvee_{i=1}^{m} E_i\right)
\]

Similarly:

\[
\sum_{i=1}^{m} \Pr(E_i) \times a_i \geq (1 - \epsilon' m) \times \Pr\left(\bigvee_{i=1}^{m} E_i\right)
\]
In conclusion, we have:

\[(1 - \epsilon'm) \Pr\left(\bigvee_{i=1}^{m} E_i\right) \leq \sum_{i=1}^{m} \Pr(E_i) \times a_i \leq (1 + \epsilon'm) \Pr\left(\bigvee_{i=1}^{m} E_i\right)\]

So, if we want the error \(\epsilon\) we should use \(\epsilon' = \epsilon/m\)
Corollary

**FPRAS for UCQs**

Let $\mathcal{R}$ be a signature, and let $Q$ be a UCQ over $\mathcal{R}$. $\text{Eval}^{\text{mrg}}(Q, \mathcal{P})$ has a multiplicative FPRAS over BID databases.
| 1 | Querying Probabilistic Databases |
| 2 | Complexity Basics               |
| 3 | Dichotomy in CQ Complexity      |
| 4 | Approximate Query Evaluation    |
| 5 | Recap                          |
## Summary

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<th>Data Rep.</th>
<th>Complexity</th>
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</thead>
<tbody>
<tr>
<td>Exact</td>
<td>CQs w/o self joins</td>
<td>Tuple-Independent</td>
<td>Hierarchical CQs in PTime; otherwise #P-hard (Suciu-Dalvi dichotomy)</td>
</tr>
<tr>
<td>Additive Approx.</td>
<td>Any PTime language</td>
<td>pc-instances</td>
<td>FPRAS (average over samples)</td>
</tr>
<tr>
<td>Relative Approx.</td>
<td>UCQs (positive RA)</td>
<td>BID-instances</td>
<td>FPRAS (Karp-Luby-Madras)</td>
</tr>
</tbody>
</table>
References I


J. S. Provan and M. O. Ball, *The complexity of counting cuts and of computing the probability that a graph is connected.*, SIAM Journal on Computing 12 (1983), no. 4, 777–788.

End of lecture 10

Query Answering in Probabilistic Databases