Database Management Systems

Course 236363

Tutorial 6: Datalog
Outline

• Datalog programs
  – Basic definitions
  – EDBs and IDBs

• Semantics
  – Logical interpretation
  – Model theoretic semantics

• Safety

• Extensions
  – Recursion
  – Negation

• Questions
Datalog Program

• Logical Programming:
  – finding solution to a set of requirements given as logical rules

• Program example:
  \[ \text{married_man}(Y) \leftarrow \text{married_to}(X, Y). \]

• Input:
<table>
<thead>
<tr>
<th>Woman</th>
<th>Man</th>
<th>Married to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sue</td>
<td>Bob</td>
<td>Sue</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bob</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ed</td>
</tr>
</tbody>
</table>

• Output:
<table>
<thead>
<tr>
<th>Married Man</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bob</td>
</tr>
</tbody>
</table>
Basic Definitions

• An **atomic formula** has the form $R(t_1, ..., t_k)$ where:
  – $R$ is a $k$-ary relation symbol
  – Each $t_i$ is either a constant or a variable

• A **Datalog rule** has the form

$$\text{head} \leftarrow \text{body}$$

where **head** is an atomic formula and **body** is a sequence of atomic formulas
  – For simplicity, we disallow constants in the head

• A **Datalog program** is a finite set of Datalog rules
EDBs and IDBs

• Datalog rules operates over:
  – **Extensional Database** (EDB) predicates
    • These are the provided/stored database relations from the relational schema
  – **Intentional Database** (IDB) predicates
    • These are the relations *derived* from the stored relations through the rules
    • Each IDB appears as a head of some rule

\[
\text{married\_man}(Y) \leftarrow \text{married\_to}(X, Y).
\]
Outline

• Datalog programs
  – Basic definitions
  – EDBs and IDBs
• Semantics
  – Logical interpretation
  – Model theoretic semantics
• Safety
• Extensions
  – Recursion
  – Negation
• Questions
Logical Interpretation of a Rule

- The rule

\[
\text{married\_man}(Y) \leftarrow \text{married\_to}(X, Y).
\]

is interpreted as the logical rule:

\[
\forall Y [\exists X [\text{married\_to}(X, Y) \rightarrow \text{married\_man}(Y)]
\]

which is equivalent to

\[
\forall Y \forall X [\text{married\_to}(X, Y) \rightarrow \text{married\_man}(Y)]
\]
Logical Interpretation of a Rule

- The rule

\[ \text{married\_man}(Y) \leftrightarrow \text{married\_to}(X, Y), \text{man}(Y). \]

is interpreted as the logical rule:

\[ \forall Y \exists X [\text{married\_to}(X, Y) \land \text{man}(Y)] \rightarrow \text{married\_man}(Y) \]

which is equivalent to

\[ \forall Y \forall X [[\text{married\_to}(X, Y) \land \text{man}(Y)] \rightarrow \text{married\_man}(Y)] \]
Semantics of Datalog Programs

- Datalog programs $P$ are defined over a schema
  - This schema contains EDB+IDB relation symbols
  - The input to $P$ is an instance $I$ over the EDB schema
  - The output of $P$ is an instance $J$ over the IDB schema
**Model-Theoretic Definition**

- We say that $J$ is a *model* of $P$ (w.r.t. $I$) if $I \cup J$ satisfies all the rules of $P$.
- We say that $J$ is a *minimal model* if $J$ does not properly contain any other model.

$$\text{married\_man}(Y) \leftarrow \text{married\_to}(X, Y).$$

<table>
<thead>
<tr>
<th>Woman</th>
<th>Man</th>
<th>Married to</th>
<th>Married Man</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sue</td>
<td>Bob</td>
<td>Sue</td>
<td>Bob</td>
</tr>
<tr>
<td></td>
<td>Ed</td>
<td></td>
<td>Bob</td>
</tr>
</tbody>
</table>

**input**

**Minimal model**
The following is also a model for $P$:

\[
\text{married\_man}(Y) \iff \text{married\_to}(X, Y).
\]

However, this model is not minimal – We can omit a tuple and still remain with a model.
Outline

• Datalog programs
  – Basic definitions
  – EDBs and IDBs

• Semantics
  – Logical interpretation
  – Model theoretic semantics

• Safety

• Extensions
  – Recursion
  – Negation

• Questions
Safety in Datalog

• What is the problem with the Datalog rule \( q(X,Y) \leftarrow p(X) \)?

• Our goal:
  – Finite output
  – Independent of the domain

• A safe rule is a rule in which
  – Every variable \( x \) is bounded, i.e., it appears in an atom \( R(\ldots,x,\ldots) \) in the body of some rule
Outline

• Datalog programs
  – Basic definitions
  – EDBs and IDBs

• Semantics
  – Logical interpretation
  – Model theoretic semantics

• Safety

• Extensions
  – Recursion
  – Negation
    • stratification

• Questions
Recursive Datalog

• Let us consider the following Datalog program:

\[
\text{Ancestor}(A,D) \leftarrow \text{Father}(A,D) \\
\text{Ancestor}(A,D) \leftarrow \text{Ancestor}(A,P), \text{Father}(P,D)
\]

• This is a recursive program
  – Ancestor is defined in terms of itself
  – Can a non-recursive program compute Ancestor?
Recursive Datalog

• The *dependency graph* of a Datalog program is the directed graph \((V,E)\) where
  – \(V\) is the set of IDB predicates (relation names)
  – \(E\) contains an edge \(R \rightarrow S\) whenever there is a rule with \(S\) in the head and \(R\) in the body

• A Datalog program is *recursive* if its dependency graph contains a cycle

• With recursion we can express *transitive closure*
  – Cannot be done without recursion
Datalog with negation

\[
\begin{align*}
\text{married\_man}(Y) & \leftarrow \text{married\_to}(X, Y). \\
\text{bachelor}(Y) & \leftarrow \text{man}(Y), \neg \text{married\_man}(Y)
\end{align*}
\]

- Input:

<table>
<thead>
<tr>
<th>Woman</th>
<th>Man</th>
<th>Married to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sue</td>
<td>Bob</td>
<td>Sue</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bob</td>
</tr>
<tr>
<td>Ed</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- If we evaluate (using chase) bachelor before married\_man was fully evaluated we can get:

<table>
<thead>
<tr>
<th>bachelor</th>
<th>married_man</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bob</td>
<td>Bob</td>
</tr>
<tr>
<td>Ed</td>
<td></td>
</tr>
</tbody>
</table>
Datalog with negation

\[
\begin{align*}
\text{married}_\text{man}(Y) & \leftarrow \text{married}_\text{to}(X, Y). \\
\text{bachelor}(Y) & \leftarrow \text{man}(Y), \neg \text{married}_\text{man}(Y)
\end{align*}
\]

• Input:

<table>
<thead>
<tr>
<th>Woman</th>
<th>Man</th>
<th>Married to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sue</td>
<td>Bob</td>
<td>Sue</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bob</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ed</td>
</tr>
</tbody>
</table>

• Additionally there are two possible minimal models:
  – The first includes

<table>
<thead>
<tr>
<th>married_man</th>
<th>bachelor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bob</td>
<td>Ed</td>
</tr>
</tbody>
</table>

  – The second includes

<table>
<thead>
<tr>
<th>married_man</th>
<th>bachelor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bob</td>
<td></td>
</tr>
<tr>
<td>Ed</td>
<td></td>
</tr>
</tbody>
</table>
Stratified Programs

• We need to change the semantics definition when we have negation
  – Intuitively, we want to first fully evaluate the relation married_man and then move to compute the relation bachelor

• We define the semantics by defining \textit{stratification}
  – Partitioning the IDB relations to “layers”
Stratified Programs

• Let $P$ be a Datalog program
• Let $E_0$ be set of EDB predicates
• A \textit{stratification} of $P$ is a partitioning of the IDBs into disjoint sets $E_1, \ldots, E_k$ where:
  – For $i=1,\ldots,k$, every rule with head in $E_i$ has body predicates only from $E_0, \ldots, E_i$
  – For $i=1,\ldots,k$, every rule with head in $E_i$ can have negated body predicates only from $E_0, \ldots, E_{i-1}$
• In general there might be more than one stratification!
• Note that all of them will lead to the same semantics.
married_man(Y) ← married_to(X, Y).
bachelor(Y) ← man(Y), ¬married_man(Y)

• In our case
  – $E_0$ includes the relation symbol married_to, man
  – $E_1$ - married_man
  – $E_2$ – bachelor
Datalog with negation

married_man(Y) ← married_to(X, Y).
bachelor(Y) ← man(Y), ¬married_man(Y)

• The evaluation
  – E₀
    | Woman     | Man     | Married to |
    | Sue       | Bob     | Sue       |
    |           |         | Bob       |
    |           |         | Ed        |
  – E₁
    | Married_man |
    | Bob        |
  – E₂
    | bachelor   |
    | Ed         |
Negation and safety

• Reminder:
  A *safe rule* is a rule in which
  – Every variable x is bounded, i.e., it appears in an atom $R(\ldots, x, \ldots)$ in the body of some rule

• Appearing in a negated atom does not bound the variable
  – The following rule is not safe
    \[
    \text{bachelor}(Y) \leftarrow \neg \text{married_man}(Y)
    \]
  – To make it safe we must bound Y, i.e.,
    \[
    \text{bachelor}(Y) \leftarrow \neg \text{married_man}(Y), \text{man}(Y)
    \]
Examples

• Write a Datalog program that defines the Binary relation never_married(x,y) where x is a woman that is not married to y

• 1\textsuperscript{st} try:
  
  never_married(x,y) \leftarrow \neg \text{married_to}(x,y)

  Incorrect!

• 2\textsuperscript{nd} try:
  
  never_married(x,y) \leftarrow \text{man}(y), \text{woman}(x), \neg \text{married_to}(x,y)
Outline

• Datalog programs
  – Basic definitions
  – EDBs and IDBs

• Semantics
  – Logical interpretation
  – Model theoretic semantics

• Safety

• Extensions
  – Recursion
  – Negation

• Questions
• Assume we have the following database
  – Event(place, time)
  – Person(id)
  – Seen(id, place, time)
• A social path between persons p and p’ is a sequence \( p_1, p_2, \ldots, p_n \) such that
  – \( p=p_1 \) and \( p’=p_n \)
  – For every \( i \) there exists an event such that both \( p_i, p_{i+1} \) have participated in
• Write a Datalog program (possibly with negation) that defines the relation Out(i,i’) such that
  – there exists a social path between i and i’
  – i and i’ haven’t participated in the same event
TogetherEvent(I, I') ← Person(I), Person(I'),
          Event(p,t), Seen(I,p,t), Seen(I',p,t)

SocialPath(I, I') ← TogetherEvent(I, I')
SocialPath(I, I') ← SocialPath(I, J),
            TogetherEvent(J, I')

Out(I,I') ← SocialPath(I,I'), ¬TogetherEvent(I, I')