Plan for Today

1. Independence / expressiveness
   • Union, projection, Cartesian product, renaming
   • Monotonicity
   • Difference
   • Selection
   • Question (e) from I-3 worksheet

2. Static optimization?

3. Intro to SQL queries

4. Slides for home reading: I4-sql up to page 64
Independence (easy)

\[ \pi \sigma \rho \times U \setminus \]

Prove that the following are independent of other five:

- Projection (\(\pi\))
- Cartesian product (\(\times\))
- Renaming (\(\rho\))
Independence (harder)

\[ \pi \not\in \rho \times \cup \setminus \]

Prove that selection (\(\sigma\)) is independent of the other five, even if restricted to selections of the type

\[ \sigma_A = '\text{const}' \]
Claim: for every \( Q \) made out of the five \( \pi \rho \times \cup \setminus \), the relation \( Q(D) \) is \textit{symmetric} in the following sense:

\(-\ t \in Q(D) \Rightarrow \text{inv}(t) \in Q(D)\)

- Tuple \( \text{inv}(t) \) is obtained from \( t \) by switching between 1s and 0s

Proof of claim: induction
- Basis: atomic query \( R \)
- Inductive: if \( Q_1(D) \) and \( Q_2(D) \) are sym. then so are:
  - \( \pi_{A_1,\ldots,A_m}Q_1(D) \) \( \rho_{A/B}Q_1(D) \)
  - \( Q_1(D) \times Q_2(D) \) \( Q_1(D) \cup Q_2(D) \) \( Q_1(D) \setminus Q_2(D) \)

Can you show an asymmetric \( Q(D) \) with \( \sigma_A=1 \)?
Independence

\[ \pi \sigma \rho \times U \]

Prove that difference (\(\setminus\)) is independent of the other five
A query $Q$ is *monotonic* if for all databases $D$, adding new tuples to $D$ cannot result in loss of tuples in $Q(D)$

$$D \subseteq E \implies Q(D) \subseteq Q(E)$$
Independence of Difference

• **Claim 1:** the query $R \setminus S$ is **not** monotonic
  – Proof: example

• **Claim 2:** every query composed by $\pi \sigma \rho \times \cup$ is monotonic
  – Proof: by induction
    • Basis: atomic query $R$
    • Inductive: if $Q_1$ and $Q_2$ are monotonic, then so are:
      – $\pi_{A_1,\ldots,A_m} Q_1$ $\sigma_{\text{cond}} Q_1$ $\rho_{A/B} Q_1$
      – $Q_1 \times Q_2$, $Q_1 \cup Q_2$

• Conclusion: difference is independent of the other five
Expressing Division $R(X,Y) \div S(Y)$

$$\pi \sigma \rho \times \cup \\setminus$$

$$\pi_X R \setminus \pi_X \left( (\pi_X R \times S) \setminus R \right)$$

Prove that division **cannot** be expressed using the primitive operators **without difference**