Database Management Systems

Course 236363

Tutorial 6:
Datalog
• **Datalog programs**
  – Basic definitions
  – EDBs and IDBs
• **Semantics**
  – Logical interpretation
  – Model theoretic semantics
• **Safety**
• **Extensions**
  – Recursion
  – Negation
• **Questions**
Datalog Program

• Logical Programming:
  – finding solution to a set of requirements given as logical rules

• Program example:

\[
\text{married\_man}(Y) \leftarrow \text{married\_to}(X, Y).
\]

• Input:

<table>
<thead>
<tr>
<th>Woman</th>
<th>Man</th>
<th>Married to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sue</td>
<td>Bob</td>
<td>Sue</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bob</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ed</td>
</tr>
</tbody>
</table>

• Output:

<table>
<thead>
<tr>
<th>Married Man</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bob</td>
</tr>
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</table>
Basic Definitions

• An *atomic formula* has the form $R(t_1, ..., t_k)$ where:
  – $R$ is a $k$-ary relation symbol
  – Each $t_i$ is either a constant or a variable

• A *Datalog rule* has the form
  
  head ← body

  where *head* is an atomic formula and *body* is a sequence of atomic formulas
  – For simplicity, we disallow constants in the head

• A *Datalog program* is a finite set of Datalog rules
• Datalog rules operates over:
  – **Extensional Database (EDB)** predicates
    • These are the provided/stored database relations from the relational schema
  – **Intentional Database (IDB)** predicates
    • These are the relations *derived* from the stored relations through the rules
    • Each IDB appears as a head of some rule

```
married_man(Y) ← married_to(X, Y).
```
Outline

• Datalog programs
  – Basic definitions
  – EDBs and IDBs

• Semantics
  – Logical interpretation
  – Model theoretic semantics

• Safety

• Extensions
  – Recursion
  – Negation

• Questions
The rule

\[
\text{married\_man}(Y) \leftarrow \text{married\_to}(X, Y).
\]

is interpreted as the logical rule:

\[
\forall Y \left[ \exists X [ \text{married\_to}(X, Y) \rightarrow \text{married\_man}(Y) ] \right]
\]

which is equivalent to

\[
\forall Y \forall X \left[ \text{married\_to}(X, Y) \rightarrow \text{married\_man}(Y) \right]
\]
Logical Interpretation of a Rule

• The rule

\[
\text{married\_man}(Y) \leftrightarrow \text{married\_to}(X, Y), \text{man}(Y).
\]

is interpreted as the logical rule:

\[
\forall Y \left[ \exists X \left[ \text{married\_to}(X, Y) \land \text{man}(Y) \right] \rightarrow \text{married\_man}(Y) \right]
\]

which is equivalent to

\[
\forall Y \forall X \left[ \left[ \text{married\_to}(X, Y) \land \text{man}(Y) \right] \rightarrow \text{married\_man}(Y) \right]
\]
Semantics of Datalog Programs

• Datalog programs P are defined over a schema
  – This schema contains EDB+IDB relation symbols
  – The input to P is an instance I over the EDB schema
  – The output of P is an instance J over the IDB schema
Model-Theoretic Definition

• We say that $J$ is a *model* of $P$ (w.r.t. $I$) if $I \cup J$ satisfies all the rules of $P$

• We say that $J$ is a *minimal model* if $J$ does not properly contain any other model

$$\text{married\_man}(Y) \leftarrow \text{married\_to}(X, Y).$$

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</tr>
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**Minimal model**

**Input**
The logical rule evaluates to true on I

\[ \forall Y \forall X \left[ \text{married_to}(X,Y) \rightarrow \text{married_man}(Y) \right] \]

However, this model is not minimal
– We can omit a tuple and still remain with a model
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• Questions
Safety in Datalog

• What is the problem with the Datalog rule
  \( q(X,Y) \leftarrow p(X) \)?

• Our goal:
  – Finite output
  – Independent of the domain

• A safe rule is a rule in which
  – Every variable \( x \) is bounded, i.e., it appears in an atom \( R(\ldots,x,\ldots) \) in the body of some rule
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  – Recursion
  – Negation
    • stratification
• Questions
Recursive Datalog

• Let us consider the following Datalog program:

\[
\begin{align*}
\text{Ancestor}(A,D) & \leftarrow \text{Father}(A,D) \\
\text{Ancestor}(A,D) & \leftarrow \text{Ancestor}(A,P), \text{Father}(P,D)
\end{align*}
\]

• This is a recursive program
  – Ancestor is defined in terms of itself
  – Can a non-recursive program compute Ancestor?
Recursive Datalog

• The *dependency graph* of a Datalog program is the directed graph \((V,E)\) where
  – \(V\) is the set of IDB predicates (relation names)
  – \(E\) contains an edge \(R \rightarrow S\) whenever there is a rule with \(S\) in the head and \(R\) in the body

• A Datalog program is *recursive* if its dependency graph contains a cycle

• With recursion we can express *transitive closure*
  – Cannot be done without recursion
Datalog with negation

\[
\text{married\_man}(Y) \leftarrow \text{married\_to}(X, Y).
\]
\[
\text{bachelor}(Y) \leftarrow \text{man}(Y), \neg \text{married\_man}(Y)
\]

• Input:

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</table>

• There are two possible minimal models:
  – The first includes
    | Married\_man | bachelor |
    | Bob         | Ed       |
  – The second includes
    | Married\_man | bachelor |
    | Bob         | Ed       |
Stratified Programs

• We need to change the semantics definition when we have negation
  – Intuitively, we want to first fully evaluate the relation married_man and then move to compute the relation bachelor
• We define the semantics by defining stratification
  – Partitioning the IDB relations to “layers”
Stratified Programs

• Let $P$ be a Datalog program
• Let $E_0$ be set of EDB predicates
• A *stratification* of $P$ is a partitioning of the IDBs into disjoint sets $E_1, \ldots, E_k$ where:
  – For $i=1,\ldots,k$, every rule with head in $E_i$ has body predicates only from $E_0, \ldots, E_i$
  – For $i=1,\ldots,k$, every rule with head in $E_i$ can have negated body predicates only from $E_0, \ldots, E_{i-1}$
• In general there might be more than one stratification!
• Note that all of them will lead to the same semantics.
married_man(Y) ← married_to(X, Y).
bachelor(Y) ← man(Y), ¬married_man(Y)

• In our case
  – \( E_0 \) includes the relation symbol married_to, man
  – \( E_1 \) - married_man
  – \( E_2 \) – bachelor
Datalog with negation

`married_man(Y) ← married_to(X, Y).`
`bachelor(Y) ← man(Y), ¬married_man(Y)`

• The evaluation

- $E_0$

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- $E_1$

<table>
<thead>
<tr>
<th>Married_man</th>
</tr>
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<td>Bob</td>
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- $E_2$

<table>
<thead>
<tr>
<th>bachelor</th>
</tr>
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Negation and safety

• Reminder:
  A *safe rule* is a rule in which
  – Every variable $x$ is bounded, i.e., it appears in an atom $R(\ldots,x,\ldots)$ in the body of some rule

• Appearing in a negated atom does not bound the variable
  – The following rule is not safe
    \[
    \text{bachelor}(Y) \leftarrow \neg \text{married\_man}(Y)
    \]
  – To make it safe we must bound $Y$, i.e.,
    \[
    \text{bachelor}(Y) \leftarrow \neg \text{married\_man}(Y), \text{man}(Y)
    \]
Examples

• Write a Datalog program that defines the Binary relation never_married(x,y) where x is a woman that is not married to y

• 1\textsuperscript{st} try:
  \texttt{never_married(x,y)\leftarrow \neg\text{married_to(x,y)}}
  \textbf{Incorrect!}

• 2\textsuperscript{nd} try:
  \texttt{never_married(x,y)\leftarrow \text{man(y)}, \text{woman(x)}, \neg\text{married_to(x,y)}}
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• Questions
• Assume we have the following database
  – Event(place, time)
  – Person(id)
  – Seen(id, place, time)

• A social path between persons p and p’ is a sequence p₁, p₂, ..., pₙ such that
  – p=p₁ and p’=pₙ
  – For every i there exists an event such that both pᵢ, pᵢ₊₁ have participated in

• Write a Datalog program (possibly with negation) that defines the relation Out(i,i’) such that
  – there exists a social path between i and i’
  – i and i’ haven’t participated in the same event
Answer

TogetherEvent(I, I') ⇐ Person(I), Person(I'), Event(p,t), Seen(I,p,t), Seen(I',p,t)

SocialPath(I, I') ⇐ TogetherEvent(I, I')
SocialPath(I, I') ⇐ SocialPath(I, J), TogetherEvent(J, I')

Out(I, I') ⇐ SocialPath(I, I'), ¬TogetherEvent(I, I')