Exercise 1
Consider the problem:

\[
\begin{aligned}
    \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} &= 0, \quad 0 < x < 1; t > 0 \\
    u(x,0) &= 4x(1-x), \quad 0 < x < 1 \\
    u(0,t) &= u(1,t) = 0, \quad t \geq 0
\end{aligned}
\]

(a) Build the explicit scheme of the problem.
(b) Build the 3-level scheme of the problem or use the Newton's linearization method.
(c) Solve the problem numerically using methods from (a) and (b) for
\[
r = \frac{k}{h^2} = 0.5; \quad h = 0.1; \quad t = 0.5; \quad t = 1.0.
\]
Compare the results obtained by (a) and (b).
Discuss and conclude.
Exercise 2

Consider the problem:

\[
\begin{aligned}
  u_t &= u_{xx} + u_{yy}, & \text{in } \Omega, & \ t > 0; \\
  u(x,y,0) &= 1, & \text{in } \Omega \\
  u(x,y,t) &= 0, & \text{on } \partial\Omega, & t > 0
\end{aligned}
\]

when \( \Omega \) is defined to be a domain between the rectangular 1x1 and the circle of radius 0.3 centered at (0.5,0.5). \( \partial\Omega \) denotes the boundary of the domain \( \Omega \).

(a) Solve the problem numerically using the explicit scheme of the problem for \( \Delta x = \Delta y = \frac{1}{50} \).

(b) (bonus)
Using the Von-Newman method, prove that the stability criteria of the scheme is given by

\[
2\left(\frac{v_x}{\alpha} + \frac{v_y}{\beta}\right) \leq 1, \quad \text{where } v_x = \frac{\Delta t}{(\Delta x)^2}; v_y = \frac{\Delta t}{(\Delta y)^2}.
\]

(c). Compare the stability condition of (b) (even if you did not prove it) with one obtained for the explicit scheme in rectangular domain.

(d). Solve the problem numerically using the ADI scheme.

Good luck!!!