Applications

• Oversampled 3D scan data

~150k triangles
~80k triangles
Applications

- Over tessellation: E.g. iso-surface extraction

Applications

- Multi-resolution hierarchies for
  - efficient geometry processing
  - level-of-detail (LOD) rendering
Applications

- Adaptation to hardware capabilities

Size-Quality Tradeoff
Problem Statement

• Given: \( \mathcal{M} = (\mathcal{V}, \mathcal{F}) \)

• Find: \( \mathcal{M}' = (\mathcal{V}', \mathcal{F}') \) such that

1. \( |\mathcal{V}'| = n < |\mathcal{V}| \) and \( \| \mathcal{M} - \mathcal{M}' \| \) is minimal, or

2. \( \| \mathcal{M} - \mathcal{M}' \| < \varepsilon \) and \( |\mathcal{V}'| \) is minimal

• Respect additional fairness criteria
  – normal deviation, triangle shape, scalar attributes, etc.

Mesh Decimation Methods

• Vertex clustering

• Incremental decimation

• Resampling

• Mesh approximation
Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
- Topology changes

- Cluster Generation
  - Uniform 3D grid
  - Map vertices to cluster cells
- Computing a representative
- Mesh generation
- Topology changes
Vertex Clustering

• Cluster Generation
  – Hierarchical approach
  – Top-down or bottom-up

• Computing a representative

• Mesh generation

• Topology changes
Computing a Representative Average vertex position

Computing a Representative Median vertex position
Computing a Representative Error Quadrics

Error quadrics

Error Quadrics

- Patch is expected to be piecewise flat
- Minimize distance to neighboring triangles’ planes
Error Quadrics

- Squared distance of point \( p \) to plane \( q \):

\[
p = (x, y, z, 1)^T, \quad q = (a, b, c, d)^T
\]

\[
dist(q, p)^2 = (q^T p)^2 = p^T (qq^T) p =: p^T Q_q p
\]

\[
Q_q = \begin{bmatrix}
a^2 & ab & ac & ad \\
ba & b^2 & bc & bd \\
cb & ca & c^2 & cd \\
da & db & dc & d^2
\end{bmatrix} = \begin{bmatrix}
E_{3\times3} & -f \\
-f^T & g
\end{bmatrix}
\]

Error Quadrics

- Sum distances to planes \( q_i \) of vertex' neighboring triangles:

\[
\sum_i dist(q_i, p)^2 = \sum_i p^T Q_{q_i} p = p^T \left( \sum_i Q_{q_i} \right) p =: p^T Q_p p
\]

- An expression of the energy using \( E, f, g \):

\[
p^T \begin{bmatrix}
E_{3\times3} & -f \\
-f^T & g
\end{bmatrix} p
\]
Error Quadrics

• Point $p^*$ that minimizes the error satisfies (simple linear least squares):
  
  $$ p = (x, y, z, 1)^T, \quad \min p^T \begin{bmatrix} E_{3x3} & -f \\ -f^T & g \end{bmatrix} p \Rightarrow $$
  
  $$ Ep = f $$

• Achieved by taking derivatives for all $(x,y,z)$.
• $E$ might not be full rank (why?)
  – Use pseudo-inverse

Comparison

average
median
error quadric
Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
  - Clusters \( p \leftrightarrow \{p_0, ..., p_n\} \), \( q \leftrightarrow \{q_0, ..., q_m\} \)
- Topology changes

Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
  - Clusters \( p \leftrightarrow \{p_0, ..., p_n\} \), \( q \leftrightarrow \{q_0, ..., q_m\} \)
  - Connect \( (p, q) \) if there was an edge \( (p_i, q_j) \)
- Topology changes
Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
- Topology changes
  - If different sheets pass through one cell
  - Can be non-manifold

Outline

- Applications
- Problem Statement
- Mesh Decimation Methods
  - Vertex Clustering
  - Incremental Decimation
  - Extensions
Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes
General Setup

- Repeat:
  - pick mesh region
  - apply decimation operator
  - Until no further reduction possible

Greedy Optimization

- For each region
  - evaluate quality after decimation
  - enqueue(quality, region)

- Repeat:
  - get best mesh region from queue
  - apply decimation operator
  - update queue
  - Until no further reduction possible
Global Error Control

- For each region
- evaluate quality after decimation
- enqueue(quality, region)

- Repeat:
  - get best mesh region from queue
  - if error < \( \varepsilon \)
  - apply decimation operator
  - update queue
- Until no further reduction possible

Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes
Decimation Operators

- What is a "region"?
- What are the DOF for re-triangulation?
- Classification
  - Topology-changing vs. topology-preserving
  - Subsampling vs. filtering
  - Inverse operation → progressive meshes

Vertex Removal

Select a vertex to be eliminated
Vertex Removal

Select all triangles sharing this vertex

Vertex Removal

Remove the selected triangles, creating the hole
Vertex Removal

Fill the hole with new triangles

Decimation Operators

- Remove vertex
- Re-triangulate hole
  - Combinatorial degrees of freedom
Decimation Operators

- Merge two adjacent vertices
- Define new vertex position
  - Continuous degrees of freedom
  - Filter along the way

Decimation Operators

- Collapse edge into one end point
  - Special case of vertex removal
  - Special case of edge collapse
- No degrees of freedom
- Separates global optimization from local optimization
Half-Edge Collapse

Half-Edge Collapse
Half-Edge Collapse

Half-Edge Collapse
Half-Edge Collapse

Half-Edge Collapse
Half-Edge Collapse

Half-Edge Collapse
Incremental Decimation

• General Setup
• Decimation operators
• Error metrics
• Fairness criteria
• Topology changes

Local Error Metrics

• Local distance to mesh
  – Compute average plane
  – No comparison to original geometry
Global Error Metrics

- Error quadrics
  - Squared distance to planes at vertex
  - No bound on true error

\[ p_i^T Q_i p_i = 0, \quad i = \{1, 2\} \]

\[ Q_3 = Q_1 + Q_2 \]

\[ v_3^T Q_3 v_3 = \min \quad < \varepsilon \quad \rightarrow \quad \text{ok} \]

Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes
Fairness Criteria

• Rate quality of decimation operation
  – Approximation error
  – Triangle shape
  – Dihedral angles
  – Valence balance
  – ...

Fairness Criteria

• Rate quality after decimation
  – Approximation error
  – Triangle shape
  – Dihedral angles
  – Valence balance
  – ...

\[
\frac{\tau_1}{e_1} < \frac{\tau_2}{e_2}
\]
Fairness Criteria

- Rate quality after decimation
  - Approximation error
  - Triangle shape
  - Dihedral angles
  - Valence balance
  - ...
Fairness Criteria

• Rate quality after decimation
  – Approximation error
  – Triangle shape
  – Dihedral angles
  – Valence balance
  – Color differences
  – ...

Fairness Criteria

• Rate quality after decimation
  – Approximation error
  – Triangle shape
  – Dihedral angles
  – Valence balance
  – Color differences
  – ...

Geometric Modeling
Mesh Decimation
**Mesh Decimation**

**Fairness Criteria**

- Rate quality after decimation
  - Approximation error
  - Triangle shape
  - Dihedral angles
  - Valence balance
  - Color differences
  - ...

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**Incremental Decimation**

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes
Topology Changes?

• Merge vertices across non-edges
  – Changes mesh topology
  – Need *spatial neighborhood* information
  – Generates *non-manifold* meshes

![Vertex Contraction](image)

![Vertex Separation](image)
## Comparison

- **Vertex clustering**
  - fast, but difficult to control simplified mesh
  - topology changes, non-manifold meshes
  - global error bound, but often not close to optimum

- **Incremental decimation with quadric error metrics**
  - good trade-off between mesh quality and speed
  - explicit control over mesh topology
  - restricting normal deviation improves mesh quality

## The Exercise

- **Do Edge-collapse with error quadrics**

- **Tasks:**
  - Compute Quadric error for each pair of vertices sharing an edge
  - Test that an halfedge collapse is valid (triangles remain valid), no intersections
  - Choose next candidate according to sum of error quadrics
  - Collapse and repeat until target mesh size achieved