Tutorial 8 – Part B

Image Enhancement via Histogram Operations

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The histogram of a given image $X$ is defined as:

$$\text{hist}_X(\alpha) = |\{(i,j)| x(i,j) = \alpha\}|$$

where

$\text{hist}_X(\alpha)$ is the number of pixels in the image $X$ of a graylevel $\alpha$. 
Image Histogram
Image Histogram

• The histogram is only a partial description of the image.

• Example:
The following two images have the same histogram.
We will consider mapping operations applied on histograms:

All the pixels in the graylevel \( \alpha \) will get a new value
\[
\alpha_{\text{new}} = g(\alpha) \quad \text{where } g \text{ is the mapping function.}
\]

- Only the pixel value is considered (and not its location).
- These are memoryless operations, as they independently consider each pixel.

- *Enhancement* is usually achieved when:
  - \( g \) is monotonically increasing.
  - The resultant histogram is
    - spread over the range \([0, 255]\).
    - Uniform.
  - Usually, the enhancement is subjectively evaluated.
Histogram Stretching

How can we visually improve the following image?
Histogram Stretching

Let us define:

- The *input*-image histogram is in the range $[L^-, L^+]$.
- The *output*-image histogram is in the range $[0, L_{new} - 1]$

$$
\alpha_{new} = \frac{\alpha - L^-}{L^+ - L^-}(L_{new} - 1)
$$

Note that $\alpha_{new}$ is not necessarily an integer. Hence, it should be rounded for the case of integer-valued graylevels.
Histogram Stretching

- What can still be improved?
Histogram Mapping via Thresholding

Original image  Noisy image  The mapping function  The recovered image
Histogram Equalization

• An attempt to equalize the histogram such that the graylevel distribution will be uniform as possible.
  – This approach is motivated by the heuristic assumption that uniform histogram will visually improve the image.

• Let’s analyze this in a continuous framework, assuming the graylevels are in the range \([0,1]\):
  – We consider the probability-density-function (PDF) of the image graylevels (i.e., the normalized histograms):
    • The original-image PDF is \(f_1(\alpha)\)
    • The processed-image PDF is \(f_2(\alpha)\)

  – We are looking for a mapping function \(\alpha_2 = g(\alpha_1)\) such that \(f_2\) is uniform, i.e., \(f_2(\alpha) = 1\) for \(\alpha \in [0,1]\).
Histogram Equalization

- Calculation of the resultant histogram:

\[
f_2(\alpha_2) = \frac{d}{d\alpha_2} F_2(\alpha_2) = \frac{d}{d\alpha_2} \int_0^{\alpha_2} f_2(\theta) d\theta =
\]

\[
\alpha_2 = g(\alpha_1) \text{ and is monotonically increasing}
\]

\[
= \frac{d}{d\alpha_2} \int_0^{\alpha_1} f_1(\theta) d\theta = \frac{d}{d\alpha_1} \frac{d\alpha_1}{d\alpha_2} \int_0^{\alpha_1} f_1(\theta) d\theta = f_1(\alpha_1) \cdot \frac{d\alpha_1}{d\alpha_2}
\]

- Let's consider the mapping function \( \alpha_2 = g(\alpha_1) = F_1(\alpha_1) \):

\[
\frac{d\alpha_1}{d\alpha_2} = \frac{d}{d\alpha_2} F_1^{-1}(\alpha_2) = \frac{1}{\frac{d}{dx} F_1(x) \bigg|_{x=F_1^{-1}(\alpha_2)}} = \frac{1}{f_1(\alpha_1)}
\]

\[
f_2(\alpha_2) = f_1(\alpha_1) \cdot \frac{d\alpha_1}{d\alpha_2} = f_1(\alpha_1) \frac{1}{f_1(\alpha_1)} = 1
\]

\( F_1 \) and \( F_2 \) are the cumulative-distribution-functions (CDFs) that correspond to \( f_1 \) and \( f_2 \), respectively.

Hence, we've got a uniform PDF.
Histogram Equalization in Practice

The corresponding discrete version of

\[ g(\alpha_1) = F_1(\alpha_1) = \int_{\theta=0}^{\alpha_1} f_1(\theta) d\theta \]

is

\[ g(\alpha_1) = P_1(\alpha_1) = \sum_{n=0}^{\alpha_1} p_1(n) \]
Before Equalization

![Image of a camera setup](image)

![Histogram of gray level vs. number of pixels](chart)
After Equalization
The Mapping Function

The mapping function (F)

The original histogram
Histogram Equalization

original  stretched  equalized
Artifacts of Histogram Equalization

• The background is usually a major portion of the image, hence affect the mapping function and gets too much graylevels.

• Noise is amplified.

• False contouring and other deteriorating details.