Statistical Data Processing (236200)

Winter 2017-18

Homework #3

Publication: December 14th, 2017
Submission: by January 4th at 13:30

Guidelines:

- Submission is in singles.
- Submit your solution to the TA mailbox at Taub floor 5.
- Matlab code should be electronically submitted via the course website.
- You may answer in Hebrew or English, in a clear-handwrite or a printed form.
- Follow the FAQ at course website, as clarifications and corrections may be published only there.

הנחיות:

ניתן להגיש ידנית.
הגשת ידנית לצל המתרגל האחראי בקומת 5.5
את קוד המטלב יש להגיש אלקטורונים או אחור הקורס.
ניתן להענות בבררות או באנגלית, בכל פרווסט.
voie לעובד אחריו בבררות. הביאה להטיקונים בקושי להנפיקי פרוסומ שכם בבלד.
**Question #1 (30 Points)**

Consider a scalar quantizer $Q(x)$ that is defined for any $x \in \mathbb{R}$.

The quantizer has $K$ representation values, $\{r_i\}_{i=1}^{K}$, and corresponding decision levels $\{d_i\}_{i=1}^{K-1}$ such that the quantization function is defined as

$$Q(x) = \begin{cases} 
  r_1, & \text{for } x < d_1 \\
  r_i, & \text{for } d_{i-1} \leq x < d_i, \quad i = 2, \ldots, K-1 \\
  r_K, & \text{for } x \geq d_{K-1} 
\end{cases}$$

Note that the quantizer defined in this question is not necessarily optimized in any particular way.

The signal $\phi_{in}(t)$, defined for $t \in [0,1)$, has the periodic extension $\phi_{in}^{(p)}(t)$ that is defined for any $t \in (-\infty, \infty)$ as

$$\phi_{in}^{(p)}(t) \triangleq \phi_{in}(t - \lfloor t \rfloor)$$

The system $\mathcal{H}\{\cdot\}$ does scalar quantization on the input values.

The system’s output for the input signal $\phi_{in}^{(p)}(t)$ is

$$\phi_{out}(t) \triangleq \mathcal{H}\{\phi_{in}^{(p)}(t)\} = Q\left(\phi_{in}^{(p)}(t)\right)$$

a. (10 Points) Is $\mathcal{H}$ a linear system? Prove.

b. (10 Points) Is $\mathcal{H}$ a shift-invariant system? Prove.
**Question #2 (30 points) – Optimal Basis for Smooth Signals**

Complete the proof of Theorem 3 (see lecture 8, slide 21) by showing the uniqueness of the Fourier basis for the case of a simple spectrum with \(0 < \lambda_1 < \lambda_2 < \lambda_3 < \cdots\).

Your proof should rely on Poincaré’s “magic trick” that appears also in the uniqueness proof given in the following paper for a general spectrum:


Your proof should be a simplified version of the proof in the above paper, as we consider here the simple spectrum defined above (and in the lecture notes).

**Question #3 (10 points) – The Discrete Fourier Transform**

Calculate the DFT (of order \(N\)) of the discrete one-dimensional signal with values defined for \(n = 0, ..., N - 1\) via

\[
x[n] = \begin{cases} 1 & \text{for } n = 0, T, ..., (c - 1)T \\ 0 & \text{otherwise} \end{cases}
\]

where \(N = cT\) for some positive integer \(c\).
Matlab Part (30 points) – Image Restoration using Filtering in The DFT Domain

General instructions:

- Figures should be appropriately titled.
- Your written report should be submitted with the dry part, whereas the code should be electronically submitted.

a. Select a grayscale image of size 512 × 512 pixels and create a deteriorated version of it by adding a constant to the graylevel values for all the pixels belonging to columns that their index is an integer multiplication of 16. Present the original image and the deteriorated one (that should have damages in the form of periodically placed vertical-lines).

b. Considering the $j^{th}$ line, what is the DFT of the interference? You may assist your solution to Question #3 above. In which DFT components the interference has a strictly positive values?

c. Restore the image from its corrupted version by applying Notch filtering in the DFT domain for each row. Compare the MSE of the deteriorated image to the MSE of the restored image. Note that in this question you should implement the DFT by yourselves (and not by using Matlab's fft function).