Statistical Data Processing – Winter 2017-18

Final Project

Project List

Presentation & Submission Date: 19.3.18 at 10:30.

Clarifications:

- Please e-mail your project preference to the course staff.

- The project descriptions given here are for general definition of the project goals, and particularly emphasize the required extensions to the material taught in class.

- In your presentations and reports you are required to show a full understanding of the material from class and its extensions.

- You are encouraged to include in your projects discussions and demonstrations further to those explicitly specified here. In turn, this may positively affect the evaluation of your project in the originality and independent-work aspects.
**Project #1: Wiener Filtering for Signal Restoration**

**Part I: Denoising**

Study the topic of Wiener filtering for denoising a signal from additive white Gaussian noise.

Present mathematical analysis and Matlab demonstrations.

Show the Wiener filter solution to this problem for synthetic images where the signal autocorrelation is exactly known (i.e., construct a random family of images and mathematically calculate its accurate autocorrelation).

In addition, design a nonlinear algorithm that will perform better than the Wiener filter.

A family of the synthetic images may be defined as follows:

Square $B \times B$ images formed according to the following four random variables:

Two graylevels $L_1, L_2$

The position of the line, $X$

The binary variable $D$ that defines the line as vertical or horizontal.

Note that $B$ cannot be too large due to computational reasons.

Start by implementing the above for the corresponding *one-dimensional problem*. 

![Diagram of Wiener filter solution](image_url)
Part II: Restoration

Extend the study of the denoising problem to consider a degradation including a linear operator and an additive noise (the degradation model is as studied in class).

Present mathematical developments (considering a sufficiently simple linear operator), and also show the corresponding empirical results (show it also for a more complicated linear operator).

Part III

In this part consider the cases above that were analyzed mathematically. Demonstrate the convergence of the empirical experiment as a function of the number of examples.
Project #2: Wiener Filtering for Denoising of Polygonal-Shaped Images

In this project you will denoise full-size corrupted images based on a statistical model of a class of small image patches.

The clean images and patches contain only two graylevels $L_1, L_2$ (these are deterministic values).

1. The class of random image patches

A patch of size $B \times B$ (it is suggested to start from trying $B = 7$) is defined by a line that intersects the patch into two parts of graylevels $L_1, L_2$. The line is defined by its radius, $\rho$, from the central pixel of the patch and an angle $\theta$:

\[
\varphi(x, y) = \begin{cases} 
L_1, & \text{if } (x - x_0) \cdot \cos \theta + (y - y_0) \cdot \sin \theta - \rho < 0 \\
L_2, & \text{otherwise}
\end{cases}
\]

where $(x_0, y_0)$ is the center of the patch (note that the latter formulation is accurate for a continuous domain, whereas you will use it for a discrete grid).

In addition, $\rho$ and $\theta$ are independent random variables, taking values from a discrete and finite set of options, i.e.,

\[
P_\rho(\rho_i) = q_i \quad \text{for } i = 1, ..., M_\rho.
\]

and

\[
P_\theta(\theta_i) = w_i \quad \text{for } i = 1, ..., M_\theta.
\]

Accordingly, this class contains $M_\theta M_\rho$ options for the patches, each is achieved by a pair of $\theta$ and $\rho$ values.

Mathematically analyze the statistical properties of this class, for various probability functions (i.e., values of $\{w_i\}_{i=1}^{M_\theta}$ and $\{q_i\}_{i=1}^{M_\rho}$), and demonstrate it in Matlab.

Formulate the second order statistics (mean, which is not necessarily zero here, and covariance) of this class and use it to construct the Wiener filter for denoising a given corrupted patch from additive white Gaussian noise.

2. Denoise a full image of arbitrary $H \times W$ size.

Consider a two-graylevel (the above $L_1, L_2$) full-sized image containing large polygonal shapes.
The denoising here will be applied in two different ways:
1. Denoise all the non-overlapping patches of the image and averaging the results.
2. Denoise all the overlapping patches of the image and averaging the results.

Example for an image:

- Repeat the experiments for various values of:
  - The graylevels $L_1, L_2$.
  - Noise variance.
  - Probability values of $\{w_i\}_{i=1}^{M_\theta}$ and $\{q_i\}_{i=1}^{M_\rho}$.

- Consider the cases above that were analyzed mathematically. Demonstrate the convergence of the empirical experiment as a function of the number of examples.

**Project #3: Self Functional Maps**

Use functional maps between the Laplacian eigenstructures of the same shape - once with a regular metric and then with the scale invariant one to classify shapes.

Prove the applicability of such a signature on the SHREC and TOSCA data sets.

Optional: Analyze the theoretical validity of such a signature.