Viewing and Projections

Based on lectures by Ed Angel
Computer Viewing

- There are three aspects of the viewing process, all of which are implemented in the pipeline,
  - Positioning the camera
    - Setting the model-view matrix
  - Selecting a lens
    - Setting the projection matrix
  - Clipping
    - Setting the view volume
The Default Camera

- Initially, the object and camera frames are the same
  - Model-view matrix is an identity
- The camera is located at origin and points in the negative z direction
- The default view volume is a cube with sides of length 2 centered at the origin
  - Default projection matrix is an identity
The Default Projection

- Default projection is orthogonal
Projections and Normalization

- Default projection in the camera frame is orthogonal
- For points within the default view volume
  \[
  x_p = x \\
  y_p = y \\
  z_p = 0
  \]
- Most graphics systems use view normalization
  - All other views are converted to the default view by transformations that determine the projection matrix
  - Allows use of the same pipeline for all views
OpenGL Orthogonal Viewing

- Ortho(left, right, bottom, top, near, far)

*near and far measured from camera*
OpenGL Perspective

- Frustum(left,right,bottom,top,near,far)
Using Field of View

- With Frustum it is often difficult to get the desired view.
- Perspective($\text{fovy}$, aspect, near, far) often provides a better interface.

\[
\text{aspect} = \frac{w}{h}
\]
Orthogonal Normalization

- Ortho(left, right, bottom, top, near, far)

normalization $\Rightarrow$ find transformation to convert specified clipping volume to default
Orthogonal Matrix

- Two steps
  - Move center to origin
    $$T\left(-\frac{(left+right)}{2}, -\frac{(bottom+top)}{2},\frac{(near+far)}{2}\right)$$
  - Scale to have sides of length 2
    $$S\left(\frac{2}{left-right}, \frac{2}{top-bottom}, \frac{2}{near-far}\right)$$

$$P = ST = \begin{bmatrix}
\frac{2}{right-left} & 0 & 0 & -\frac{right-left}{right-left} \\
0 & \frac{2}{top-bottom} & 0 & -\frac{top+bottom}{top-bottom} \\
0 & 0 & \frac{2}{near-far} & \frac{far+near}{far-far} \\
0 & 0 & 0 & 1
\end{bmatrix}$$
Final Projection

- Set $z = 0$
- Equivalent to the homogeneous coordinate transformation

$$\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$$

- Hence, general orthogonal projection in 4D is

$$P = M_{\text{orth}}ST$$
Oblique Projections

- The OpenGL projection functions cannot produce general parallel projections such as

- However if we look at the example of the cube it appears that the cube has been sheared

- Oblique Projection = Shear + Orthogonal Projection
General Shear

- side view

- top view
Shear Matrix

- xy shear (z values unchanged)

\[ \mathbf{H}(\theta, \phi) = \begin{bmatrix} 1 & 0 & -\cot \theta & 0 \\ 0 & 1 & -\cot \phi & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

- Projection matrix

- General case:

\[ \mathbf{P} = \mathbf{M}_{\text{orth}} \mathbf{H}(\theta, \phi) \]

\[ \mathbf{P} = \mathbf{M}_{\text{orth}} \mathbf{STH}(\theta, \phi) \]
Equivalency
Simple Perspective

- Consider a simple perspective with the COP at the origin, the near clipping plane at $z = -1$, and a 90 degree field of view determined by the planes $x = \pm z, y = \pm z$.
Perspective Matrices

- Simple projection matrix in homogeneous coordinates

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & -1 & 0 \\
\end{bmatrix}
\]

- Note that this matrix is independent of the far clipping plane
Perspective warp

\[
N = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \alpha & \beta \\
0 & 0 & -1 & 0 \\
\end{bmatrix}
\]

After perspective division, the point \((x, y, z, 1)\) goes to:

\[
x^p = \frac{x}{z}
\]

\[
y^p = \frac{y}{z}
\]

\[
z^p = -\left(\alpha + \frac{\beta}{z}\right)
\]

Which projects orthogonally to the desired point regardless of \(\alpha\) and \(\beta\).
Picking $\alpha$ and $\beta$

If we pick

$$\alpha = \frac{\text{near} + \text{far}}{\text{far} - \text{near}}$$
$$\beta = \frac{2\text{near} \cdot \text{far}}{\text{near} - \text{far}}$$

The near plane is mapped to $z = -1$
The far plane is mapped to $z = 1$
and the sides are mapped to $x = \pm 1, y = \pm 1$
Hence the new clipping volume is the default clipping volume