Subdivision Schemes
What is Subdivision?

- Subdivision is a process in which a poly-line/mesh is recursively refined in order to achieve a smooth curve/surface.

- Two main groups of schemes:
  - Approximating - original vertices are moved
  - Interpolating – original vertices are unaffected
Why Subdivision?

Frame from “Geri’s Game” by Pixar
Why Subdivision?

- LOD
- Compression
- Smoothing

52Kb

13.3Mb

424Kb

1Kb
Corner Cutting
Corner Cutting
Corner Cutting
Corner Cutting
Corner Cutting
Corner Cutting
Corner Cutting
Corner Cutting

control point

limit curve

control polygon
Corner Cutting

control point

limit curve

control polygon

This result is a quadratic B-spline curve. Known as Chaikin scheme.
Cubic Corner Cutting
Cubic Corner Cutting
Four-Point Scheme
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- Control point
- Control polygon
- Limit curve
Subdivision Curves

Non interpolatory subdivision schemes
- Corner Cutting

Interpolatory subdivision schemes
- The 4-point scheme
Basic Concepts of Subdivision

- **Subdivision curve** generated by repeatedly applying a subdivision *operator* to given polygon
- Initial polygon - *control polygon*
- Central questions:
  - **Convergence**: Given a subdivision operator and a control polygon, does the subdivision process converge?
  - **Smoothness**: Does the subdivision converge to a smooth curve?
  - **Singular points**: Does the subdivision result in a finite set of singular points?
  - **Derivatives**: How can one compute derivatives of subdivision curves/surfaces?
Subdivision Surfaces

- At each iteration
  - Refine mesh
  - Increase number of vertices

- Mesh vertices converge to a limit surface
  - After infinite number of subdivision steps

- Every subdivision method has:
  - A method to refine the mesh connectivity
  - Rules to calculate location of new vertices
    - And old if they are effected

- A scheme always consists of 2 main parts:
  - A method to generate the **topology** of the new mesh.
  - Rules to determine the **geometry** of the vertices in the new mesh.
Catmull & Clark Subdivision Scheme

- In each iteration:
  - For each face of the mesh, add a face point.
    - Usually at the centroid of the face.
  - For each edge, add an edge point
    - Average of edge points and two neighboring face points
  - Connect all face points to edge points
  - Move original points to a weighted average of adjacent points
Triangular Subdivision

- Works only for triangle meshes

- Every triangle replaced by 4 new triangles

- Two kinds of new vertices:
  - Green vertices associated with old edges
  - Yellow vertices associated with old vertices
Loop Subdivision

- New vertex is convex combination of old vertices
- List of weights called subdivision mask or stencil
  - Rule for new yellow vertices
    \( (n - \text{vertex valence}) \)
  - Rule for new green vertices

\[
\begin{align*}
\omega_n &= \frac{64n}{40 - \left(3 + 2 \cos \left(\frac{2\pi}{n}\right)\right)^2 - n}
\end{align*}
\]
Example
The Limit Surface

- Limit surfaces of Loop’s subdivision is $C^2$ almost everywhere.
- Finite set of singular locations where the surface is $C^1$. 
Butterfly Subdivision

- Interpolatory scheme
- New yellow vertices inherit location of old vertices
- New green vertices computed by following stencil:
Example
The Limit Surface

- Limit surfaces of Butterfly subdivision are $C^1$, but do not have second derivative.
Comparison
Properties

- Require regular connectivity (valence 6) to work well
- Easy to implement
- Local support
- Allow LOD
- Continuous

Drawbacks
- Not always intuitive
- Can have artifacts
- Sometimes difficult to control