Color Theory
What is Color?

- Physical definition – visible electro-magnetic energy
  - Different wavelength – different color
  - Small portion of the electro-magnetic spectrum
    - Pure *monochromatic* colors
      = wavelengths 390nm (violet) to 750nm (red)
How to Represent Color?

- Use the wavelength?
  - Doesn’t span all colors
  - Hard to generate monochromatic colors
  - Color is a visual **perceptual** property
    - Depends on the observer!


More like: roses are yellow, violets are blue
Human Color Perception

More on that later…
Human Color Perception

- Cells in the retina – rods (lavender) and cones (red)

- Cones = day vision, rods = night vision
  - More or less…

- Why are cockpit lights red?
Human Color Perception

- Cone is a photoreceptor cell
- Three types of cones: respond to different region in the spectrum

Bird with 4 types of cones

Color blind humans?
Color Representation

- Can represent colors using a small number of primaries

- Same way the retina does

![Graph showing color representation with wavelengths and colors](image-url)
Visible Color

- Eye can perceive other colors as combination of several pure colors
- Most colors may be obtained as combination of small number of *primaries*
- Output devices exploit this

![Graph showing visible light spectrum with peaks at 580, 520, and 700 nm]
Guild & Wright Experiment
Color Matching Functions

\[ \bar{r} (\lambda) \]
\[ \bar{g} (\lambda) \]
\[ \bar{b} (\lambda) \]
Reproducing a Color

- Given a distribution $I(\lambda)$
- The \textit{tristimulus values}:
  - $R = \int_0^\infty I(\lambda)\bar{r}(\lambda)$
  - $G = \int_0^\infty I(\lambda)\bar{g}(\lambda)$
  - $B = \int_0^\infty I(\lambda)\bar{b}(\lambda)$
Some Crayola Colors

- Black, Brown, Orange, Violet, Blue, Green, Red, Yellow, Blizzard Blue, Laser Lemon, Screamin’ Green, Wild Watermelon, Green, Mulberry, Raw Umber, Burnt Orange, Goldenrod, Navy Blue, Sepia, Cadet Blue, Indian Red, Plum, Sky Blue, Apricot, Gold, Orange, Silver, Bittersweet, Gray, Spring Green, Tan, Green Blue, Orchid, Thistle, Blue Green, Green Yellow, Periwinkle, Turquoise Blue, Blue Violet, Lemon Yellow, Pine Green, Violet (Purple), Brick Red, Magenta, Prussian Blue, Violet Blue, Mahogany, Violet Red, Burnt Sienna, Maize, Red Orange, White, Carnation Pink, Maroon, Red Violet, Yellow, Cornflower, Melon, Salmon, Yellow Green, Flesh, Olive Green, Sea Green, Yellow Orange, Asparagus, Macaroni and Cheese, Razzmatazz, Timber Wolf, Cerise, Mauvelous, Robin's Egg Blue, Tropical Rain Forest, Denim, Pacific Blue, Shamrock, Tumbleweed, Granny Smith Apple, Purple Mountain's Majesty, Tickle Me Pink, Wisteria Almond, Canary, Fern, Pink Flamingo, Antique Brass, Caribbean Green, Fuzzy Wuzzy Brown, Purple Heart, Banana Mania, Cotton Candy, Manatee, Shadow, Beaver, Cranberry, Mountain Meadow, Sunset Orange, Blue Bell, Desert Sand, Outer Space, Torch Red, Brink Pink, Eggplant, Pig Pink, Vivid Violet, Electric Lime, Purple Pizzazz, Razzle Dazzle Rose, Unmellow Yellow, Magic Mint, Radical Red, Sunglow, Neon Carrot, Chartreuse, Ultra Blue, Ultra Orange, Ultra Red, Hot Magenta, Ultra Green, Ultra Pink, Ultra Yellow, Atomic Tangerine, Hot Magenta, Outrageous Orange, Shocking Pink
CIE Chromaticity Diagram
(1931)

- Universal standard (Commission Internationale de l’Eclairage = International Commission on Illumination)

\[ r = \frac{R}{R + G + B} \]

\[ g = \frac{G}{R + G + B} \]

- What did we lose by normalizing?

- What colors are not in the diagram?
CIE Chromaticity Diagram
(1931)

- Visible colors contained in horse-shoe region
  - Note the domain of the axes

- Pure colors (*hues*) located on boundary of the region
The CIE Diagram

- Color “white” is point \( W=(1/3,1/3) \)

- Any visible color \( C \) is blend of a hue \( C' \) and \( W \)

- Purity of color measured by its saturation:
  \[
  \text{saturation}(C') = \frac{d_1}{d_1 + d_2}
  \]

- When does \( \text{Saturation}(C) = 1 \)? \( =0 \)?

- *Complement* of \( C \) is (the unique) other hue \( D \) on line through \( C' \) and \( W \)

- Any line through white defines complementing colors
Image Enhancement

- increase the saturation of the colors
- move them towards the boundary of the visible region

unsaturated  saturated
The RGB Color Model

- Common in describing *emissive* color displays
- Primaries are Red, Green and Blue
- Color (including intensity) described as combination of primaries
**The RGB Color Model**

The RGB Color Model is a model for representing colors by combining red, green, and blue light. The color of an object is determined as a function of the relative intensities of these three primary colors.

The RGB color model may be expressed as:

\[ Col = rR + gG + bB \]

where \( r, g, b \in [0, 1] \) are the intensities of red, green, and blue, respectively.

- **Yellow** = Red + Green: (1, 1, 0)
- **Cyan** = Green + Blue: (0, 1, 1)
- **Magenta** = Red + Blue: (1, 0, 1)
- **White** = Red + Green + Blue: (1, 1, 1)
- **Gray** = 0.5 Red + 0.5 Blue + 0.5 Green: (0.5, 0.5, 0.5)

The main diagonal of the RGB cube represents shades of gray.
RGB Sensors

Bayer pattern in digital cameras
Beyer Pattern
Not every color output device is capable of generating all the visible colors in the CIE diagram

Usually a color is generated as an affine combination of three primaries R,G,B

The possible colors are bounded by a triangle (the *gamut*) in XYZ space, whose vertices are R,G,B
  - Why triangular?
The CMY Color Model

- Used mainly in color printing, where the primary colors are subtracted from the background white.
- Cyan, Magenta and Yellow primaries are the complements of Red, Green and Blue.
- Primaries (dyes) subtracted from white paper which absorbs no energy.
  - Red = White-Cyan = White-Green-Blue \((0, 1, 1)\)
  - Green = White-Magenta = White-Red-Blue \((1, 0, 1)\)
  - Blue = White-Yellow = White-Red-Green \((1, 1, 0)\)
  - \((r,g,b) = (1-c,1-m,1-y)\)
color separation without black

color separation with black
The YIQ Color Model

- Human eye more sensitive to changes in luminance (intensity) than to changes in hue or saturation
- Luminance useful for displaying grayscale version of color signal (e.g. B&W TV)
- Luminance $Y$ – affine combination of $R,G,B$  \[ Y \in [0,1] \]
- $I$ & $Q$ – null blend (zero sum) of $R,G,B$
- Conversion

\[
(y,i,q) = (r,g,b) \begin{bmatrix}
0.30 & 0.60 & 0.21 \\
0.59 & -0.28 & -0.52 \\
0.11 & -0.32 & 0.31 \\
\end{bmatrix}
\]

- Green component dominates luminance value

Limits:
- $I \in [-0.6,0.6]$
- $Q \in [-0.52,0.52]$
The HSL Color Model

- People naturally mix colors based on Hue, Saturation and Luminance
- Resulting coordinate system is cylindrical
- H – angle around axis
- S ∈ [0,1] – distance from axis
- L ∈ [0,1] – distance from apex
RGB to HSL

- L – lightness, distance from black
- S – Saturation, distance from diagonal
- H – Hue, direction where vector is pointing
The Lightness Value

CIELAB $L^*$

HSV value $V$

HSL lightness $L$
The Importance of Perception
The Importance of Perception
The Importance of Perception
The Importance of Perception
The Importance of Perception

- Spinning dancer
The Importance of Perception

- Back to the dress...

http://www.nytimes.com/interactive/2015/02/28/science/white-or-blue-dress.html?_r=0
Impossible Colors

- What are the “colors” outside the range?
  - Colors that cannot be seen normally by the human eye
  - E.g. hyper saturated

- Can be “seen” temporarily by using visual illusions
Color Models

Summary

- Demo
- Application is important
  - Color picking – axes should be independent
    - http://www.colorpicker.com/
  - Image analysis – distances should be perceptually meaningful
Color Quantization

Most images do not cover all of RGB space
Indexed Color

replace \((R,G,B)\) values with an index to a color table
Color Quantization

Applications

- Displays with limited color resolution

ZX Spectrum (1988)
1-bit color for 8x8 blocks

These days - Embedded displays

- Image compression
Color Quantization

- 256 colors
- 64 colors
- 16 colors
- 4 colors

Lighthouse demo
Color Quantization

True color = 24bpp

256 colors = 8bpp
Color Quantization

- High-quality color resolution for images - 8 bits per primary = 24 bits = 16.7M different colors

- Reduce number of colors – select subset (colormap/pallete) and map all colors to the subset

- Used for devices capable of displaying limited number of different colors simultaneously. E.g. an 8 bit display.
Color Quantization Issues

- How are the representative colors chosen?
  - Fixed representatives, image independent - fast
  - Image content dependent - slow

- Which image colors are mapped to which representatives?
  - Nearest representative - slow
  - By space partitioning - fast
Standard Color Palettes

216 colors

16 colors
Color Quantization

256 colors

uniform

median-cut

8 colors
Choosing the Representatives

uniform quantization to 4 colors

large quantization error

image-dependent quantization to 4 colors

small quantization error
Uniform Quantization

- Fixed representatives - lattice structure on RGB cube
- Image independent - no need to analyze input image
- Some representatives may be wasted
- Fast mapping to representatives by discarding least significant bits of each component

![Uniform Quantization Diagram]

uniform quantization to 4 colors

large quantization error
Uniform Quantization

8 bits RED 8 bits GREEN 8 bits BLUE

101100100110001101101100 011000011011011101 101101100

178 99 108

1010110 1

173 index into color table
Median-Cut Quantization

- Image colors partitioned into \( n \) cells, s.t. each cell contains approximately same number of image colors

- Recursive algorithm

- Image representatives - centroids of image colors in each cell

- Image color mapped to rep. of containing cell
  - not necessarily nearest representative (example?)

image-dependent quantization to 4 colors

small quantization error
Median-Cut Quantization

input

4 colors

16 colors

256 colors

256 colors (wrong palette)

More examples
Dithering

- Newspapers have only 1 bit per color, black or white. How do they display shades of gray?
Binary Dithering

- Improve quality of quantized image by distributing quantization error.
Binary Dithering – B&W

- Threshold – map upper half of gray-level scale to white and lower half to black

- Each pixel produces some quantization error
- Distribute quantization error – use local threshold
- Use matrix of thresholds for each $n \times n$ pixels

If $I(i, j) > \frac{1}{2}$ then
$$I(i, j) = 1;$$
else
$$I(i, j) = 0;$$

If $I(i, j) > M(i \mod n, j \mod n)$ then
$$I(i, j) = 1$$
else
$$I(i, j) = 0$$
end;
Ordered Dither Matrix

- For four level input use $2 \times 2$ matrix
  
  \[
  \begin{bmatrix}
  1 & 1 \\
  1 & 1 \\
  \end{bmatrix}
  \]
  
  threshold

  \[
  \begin{bmatrix}
  0 & 2 \\
  3 & 1 \\
  \end{bmatrix}
  \]
  
  dither

- For 16 level input $4 \times 4$ matrix

  \[
  \begin{bmatrix}
  0 & 8 & 2 & 10 \\
  12 & 4 & 14 & 6 \\
  3 & 11 & 1 & 9 \\
  15 & 7 & 13 & 5 \\
  \end{bmatrix}
  \]

- Can be generalized recursively to $2^k \times 2^k$ matrix
Quantization + Dithering

- uniform
- uniform + dithering
- median-cut
- median-cut + dithering

8 colors
Error diffusion
Error Diffusion

- Reduce quantization error by propagating accumulated error from pixel to (some of) its neighbors
  - in scanline order before thresholding

**FloydSteinberg(I)**

For x := 1 to XMax do
  For y := 1 to YMax do
    err := I(x,y)-(I(x,y)>128)*256;
    I(x,y) := (I(x,y)>128)*256;
    I(x+1,y) := I(x+1,y)+err*7/16;
    I(x-1,y+1) := I(x-1,y)+err*3/16;
    I(x,y+1) := I(x,y+1)+err*5/16;
    I(x+1,y+1) := I(x+1,y+1)+err*1/16;
  end
end
Error Diffusion (cont’d)