Motivation

- In edge detection, which smoothing should we make?
- Good Smoothing - on regions from the same object
  - Lower noise, less false details
- Bad smoothing - on regions of different objects
  - Less true details
Level of details

Objects\Images are described in different levels of detail

Which level of detail is desirable ?
• For objects - maximum detail is not feasible or needed
• For images - detail is limited by pixel size, blurring
• Image based object detail depend on distance+

Which level of image detail is used in computer vision ?
Some approaches:
- Use maximal of details available
- Use the “best” level of detail
- Use several levels of details.

Scale

• Scale - two related meanings
  - the term used to described level of details.
  - Also the term used to describe magnification
• Multiscale approach - uses several scales
• Scale-space - L(x,y,t) - A redundant representation in (continuous) set of scales.

Before using multiscale ... a more fundamental question:
How to generate a version, with specific scale detail ?
How to reduce details?

- Best - using semantic descriptors
- Possible - using image data

Two ways to reduce detail with image data

- By smoothing with a filter
- By an iterative local process:
  \[ L(x, t+1) = \frac{1}{|N_j|} \sum_{j \in N_j} L(x,t) \]
  \[ L(x, t+1) - L(x, t) = \frac{1}{|N_j|} \sum_{j \in N_j} L(x,t) - \frac{|N_j|}{|N_j|} L(x, t) \]

- Discrete heat equation
- Continuous version

Intuitively: How it behaves on a constant, maxima, edge?

How to reduce details? (2)

What would be desirable properties of an image-based detail reduction process \( R(t) \)

- Uniform (Space invariance)
- Graded according to a parameter \( t \)
  \[ R(t)L(x,y,0) \rightarrow L(x,y,t) \]
- “Causal” - No “new details” are generated by \( R(t) \)
-Repeated detail level reduction gives a legal reduced level representation - Semi-group property

Which smoothing/local process satisfies the demands?
The semigroup property

Semigroup - a set with one operation “+” satisfying

- Closure: x,y in G \rightarrow x+y in G
- Associativity: (x+y)+z = x+(y+z)
- (Not needed: identity, inverse elements)

The semigroup property implies, e.g. that

If \ L(x,y,t1) = R(t1)L(x,y,0)  \\
L'(x,y) = R(t2)L(x,y,t1)

Then \ L'(x,y) \ may \ be \ written \ as \ R(t3) \ L(x,y,0) \ for \ some \ t3

Benefits: imaging compensation, computational
Note: smoothing does not necessarily have the property.

The causality property

Causality: no detail are added by detail reduction process

What is a detail?

- The term is qualitative
- Possible 1D quantifications: #level crossing, extrema

Example.

- Scale space “fingerprints” diagram
  - No minima - no detail is “born” with increasing scale.
Linear scale space

Suppose all 4 demands + Linearity are required. Then we can find a necessary condition on the process that produces a reduced detail version $L(x,t)$ from $L(x,0)$

Let $L(x,t)$ be the 1D t-detailed “image”

Consider a local maxima (with respect to $x$) of $L(x,t)$

It is a local maxima, implying

$$\frac{\partial^2 L(x,t)}{\partial x^2} < 0$$

By causality $L(x,t)$ cannot increase with $t$. Otherwise we get new level-crossings.

Linear scale space

Maxima (or minima) can occur anywhere.

$$\frac{\partial L(x,t)}{\partial t} = a(x,t,L) \frac{\partial^2 L(x,t)}{\partial x^2} \quad a(x,t,L) \geq 0$$

This is a differential equation that tells us something (weak) about the function $L(x,t)$.

Linearity+space invariance $\Rightarrow a(x,t,L) = a(t)$

Change scale parameter (monotonically) to $s$ satisfying

Yields another function $L(x,s)$

Satisfying

The heat equation

$$\frac{\partial L(x,s)}{\partial s} = \frac{1}{2} \frac{\partial^2 L(x,s)}{\partial x^2}$$
The heat equation

\[
\frac{\partial L(x,s)}{\partial s} = \frac{1}{2} \frac{\partial^2 L(x,s)}{\partial x^2}
\]

A differential equation called the heat/diffusion eq. and usually describes a process evolution over time \( s \).

Intuitive interpretation:
How it behaves on a constant, linear, maxima, edge?

The heat equation+Linear scale space

This equation has an analytic solution (which is rare)

\[
L(x,s) = \frac{1}{2\pi s} e^{-x^2/2s^2} \ast L(x,0)
\]

Therefore, a linear scale space is created by
- Convolution with Gaussian
- Running heat eq. with input image as initial condition.

Satisfies all demands: Linearity and space invariance, graded detail reduction, semigroup property, causality.
Linear scale space in images

A linear scale space is created by Gaussian Convolution

\[
L(x, y, s) = \frac{1}{2\pi s} e^{-\left(\frac{x^2+y^2}{2s^2}\right)} L(x, y, 0)
\]

- Causality is not always satisfied in the 2D case.
  Reason: we found only a necessary condition.
  - The Ridge example.

- Other advantages of Gaussians (discussed):
  intuitively correct, symmetric, separable.
Non-Linear scale spaces

A Gaussian Convolution (linear scale space) blurs & moves edges, far from semantic detail reduction.

Non-linear versions try to solve these problems

- Non-isotropic filters
- Nonlinear versions of the heat equation
  (roughly) \[ \frac{\partial L(x,y,t)}{\partial t} = \alpha(\|\nabla L\|) \nabla^2 L(x,y,t) \]
- Convolve with a modified (Gaussian) filter so that it weights similarity between value pixels:
  Bilateral, Non-local means filters.
  \[ BF[I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_x}(\| p - q \|) G_{\sigma_y}(\| I_p - I_q \|) I_q \]
Non-linear smoothing

How not to blur edges? One (non-perfect) method: Bilateral Filtering

$$\text{BF}[I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_s} (||p - q||) G_{\sigma_r} (||I_p - I_q||) I_q$$

Varying the Range Parameter

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<th>$\sigma_s$</th>
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<th>$\sigma_s = \infty$ (Gaussian blur)</th>
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 multiscale descriptions (short)

- The full scale space is a multi-scale description

- Smoothing is a low pass filters,
- Smoothed images may be sampled more coarsely.
- Pyramids - efficiently sampled discrete scale space

Applications of Scalespace & Pyramids

- Efficient search for fixed scale sub-image matches
  - Run initial search is coarse scale (small images)
  - Test only the good matches in fine scale
- Detection of objects under unknown scale
  - Compare a model subimage with different levels of a pyramids with graded smoothing & scaling
- Graphics and Image processing
  - e.g. Use Laplacian Pyramids for representation & blending
- Edge detection using feature tracking - decide on edge with support from several scales but localize it from the finest one.
- Local descriptors
  - Use several derivatives in different direction, order, scale.