236861 Numerical Geometry of Images

Tutorial 11

Active Contours

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Active Contours - introduction

- *Active contours* is a catch-all name for finding the curve that best segments an image.
- This is known as *segmentation*.
- Segmentation is highly related to tracking.
- In 3D - *active surfaces*. 
Region-based vs. Edge-based functionals

Many segmentation functionals can be divided into two groups

▶ Edge-based - the object to be segmented should have its boundary visible in the image, as some sort of prominent edge.

▶ Region-based - the region of the object in the image should have a different statistic in some feature space, compared to its surroundings.

Often, these functionals are combined with information on the shape of the region being extracted.
Examples for functionals

▶ Edge-based segmentation - Snakes (Terzopolous-Kass-Witkin ’87), Geometric active contours (Caselles-Catte-Coll-Dibos ’93, Malladi-Sethian-Vemuri ’95), Geodesic active contours (Caselles-Kimmel-Sapiro ’98).
▶ Region-based - (Cohen-Bardinet-Ayache ’93), Chan-Vese (’01), Bhattacharyya (Freedman-Zhang ’04, Rathi-Michailovich-Tannenbaum ’06).

Various shape priors are available:

▶ Geometry-based - Curvature based (Sethian ’85, Osher-Sethian ’88), affine curvature based (Angenent-Sapiro-Tannenbaum ’98) and many more.
▶ Shape-Spaces - Leventon-Grimson-Faugeras ’00, Cremers-Tischhäuser-Weickert-Schnörr ’02, and many more.
▶ Projective geometry based - Damberville-Sandhu-Yezzi-Tannenbaum ’08, Sandhu-Damberville-Yezzi-Tannenbaum ’08.
The *snakes* model try to segment the image based on the following energy:

\[ E_{\text{snake}} = E_{\text{int}} + E_{\text{ext}} \]

where

\[ E_{\text{int}} = \int \alpha |c'|^2 + \beta |c''|^2 \, ds \]

and, as one simple example,

\[ E_{\text{ext}} = -|\nabla I|^2 \]

Optimization is done using splines.
Geometric active contours

- Geometric active contours attempt to segment an object based on its edges, in a level-set framework.
- The initial contour is chosen to include the object.
- The contour evolves according to

\[ c_t = g(I)\kappa N \]

- where \( g(\cdots) \) is a function which should drop to zero at edges.
- The contour evolution tends to smooth the contour, if no other information is available.
- The contour according to this evolution will shrink to a point. Hence, a balloon force (Cohen '91) may be added

\[ c_t = (g(I)\kappa - \beta)N \]
Geodesic active contours

- However the choice of a balloon force is arbitrary.
- It is not clear if we actually minimize some functional, and the global minimizer is not clear either.
- The geodesic active contour tries to remedy this by minimizing the following weighted length functional:

\[
\int_0^{L(c)} g(I) ds
\]

- \( g(I) \) constitutes an (inverse) edge indicator. For example,

\[
g(I) = \frac{1}{\sqrt{|\nabla I|^2 + \epsilon}}
\]
Geodesic active contours (cont.)

Figure: Left: an image. Right: An example $g$ function

- The functional states that curves segmenting the object should try and surround it with a minimal weighted arclength.
This can be given a physical interpretation: We are looking for the trajectory of a particle on a map, where the potential energy at each point is $-\lambda g(I)^2$, and we assume the particle’s trajectory should form a closed simple curve.

The potential energy of the particle is given by

$$U(c) = -\lambda g(I)^2$$

From physics, the Hamiltonian will be:

$$\mathcal{H}(c) = \frac{m}{2} |c'|^2 + U(c)$$

and the Lagrangian, or difference between kinetic and potential energy, is

$$\mathcal{L}(c) = \frac{m}{2} |c'|^2 - U(c)$$

This gives us the classical approach of snakes.
The E-L equation will simply state conservation of energy for this particle under the chosen trajectory. The trajectory chosen should correspond to the Hamiltonian.

Maupertuis' principle: Curves in Euclidean space which are extremal corresponding to the Hamiltonian, and have an energy level $E_0$, are geodesics with respect to the metric

$$g_{ij} = 2m(E_0 - \mathcal{U}(c)) \delta_{ij}$$
The initial energy level $E_0$ is arbitrary.

We now choose $E_0 = 0$ (for an ideal edge we want $E_{ext} = E_{int} = 0$), and obtain

$$
\min \int_0^1 \sqrt{\lambda 2 mg(I)^2 |c'|} \, dq = \min \int_0^1 \sqrt{2m \lambda g(I) |c'|} \, dq = \sqrt{2m \lambda} \min \int_0^{L(c)} g(I) \, ds
$$

Which is an intrinsic, Euclidean-invariant functional with no need for additional parameters.

The resulting curve evolution is given by

$$
c_t = (g(I) \kappa - \langle \nabla g, N \rangle) \, N
$$
Note the geodesic interpretation also works for parts of the curve (Cohen and Kimmel, ’97).

**Figure:** Segmentation by geodesics. Left: The metric $\sqrt{g}$. Right: The resulting geodesic.
Geodesic active contours (example)
Two main techniques are available for efficiently implementing the geodesic active contours model:

- **Narrow band** (Chopp ’93, Adalsteinsson and Sethian ’95) methods compute the levelset function on only part of the image, where relevant.
- Semi-implicit schemes allow us to take large time steps and converge faster.
- Multigrid methods may also be used (Kenigsberg et. al. ’04, Papandreou and Maragos ’07).

Narrow band and splitting schemes may be combined! (Goldenberg, Kimmel, Rivlin, Rudzsky ’01)
The Mumford-Shah model

- Looking only locally at the gradient for segmentation is quite limited, in terms of basin of attraction and robustness to noise. Region statistics give us a lot more information.

- The Mumford-Shah model of image description (’89) partitions the image into:
  - Smooth parts, which we can approximate by smooth functions, and
  - A small amount of edges.

- The resulting functional is

\[
\alpha \int_{\Omega \setminus \Gamma} |\nabla u|^2 d\Omega + \beta \int_{\Omega} |u - u_0|^2 d\Omega + \mathcal{H}^{N-1}(\Gamma)
\]

- where the term \( \mathcal{H}^{N-1}(\Gamma) \) denotes the measure of boundary curves, at which discontinuities are allowed.

- The functional is defined both in terms of a 2-dimensional function, and a set of 1-dimensional curves.

- The optimization is not trivial.
Among other schemes, a well-known scheme has been suggested by Ambrosio and Tortorelli ('92) which replaces the discontinuities with an indicator function \( v \)

\[
\int_{\Omega} \left[ \alpha (1 - v)^2 \| \nabla u \|^2 + \beta |u - u_0|^2 + \rho \| \nabla v \|^2 + \frac{v^2}{2\rho} \right] d\Omega
\]

With optimization carried out on both \( u \) and \( v \) (EL’s are given in their papers).

One term states the measure of the discontinuity.

Another term is a viscosity term, favoring smooth solutions.

Taking \( \rho \to 0 \) converges to the original MS problem in a \( \Gamma \)-convergence process.

Many other schemes are available.
Active Contours Without Edges

- Chan and Vese ('00) took the Mumford-Shah model and used it to create a region-statistics based segmentation algorithm.
- Similar to a more specific, single-region, approach presented by Cohen et. al. in '93.
  - The image is assumed to be made of an object and a background, replacing the \( H^{N-1}(\Gamma) \) term with the length of a closed curve.
  - Inside and outside the object, the image intensity is assumed to be a Gaussian around a certain mean.
- This suggests a simplified model of the image, where the object has color \( \mu_1 \) and the background has color \( \mu_2 \), and a separating contour is assumed to be closed and simple.
- The resulting functional is

\[
\int_{R} (I - \mu_1)^2 \, d\Omega + \int_{\Omega \setminus R} (I - \mu_2)^2 \, d\Omega + \lambda \int \, ds
\]
- Optimization is done on both discrete parameters $\mu_1, \mu_2$, and on the levelset function.
- The levelset formulation involves a smoothed Heaviside approximation $H_\epsilon(\phi)$

\[
\int_R (I - \mu_1)^2 H_\epsilon(\phi) + (I - \mu_2)^2 (1 - H_\epsilon(\phi)) + \lambda \delta_\epsilon(\phi) \|\nabla \phi\| d\Omega
\]
The resulting PDE is

\[ \phi_t = \left[ \lambda \nabla^T \left( \frac{\nabla \phi}{|\nabla \phi|} \right) - \left( (I - \mu_1)^2 - (I - \mu_2)^2 \right) \right] \delta \epsilon(\phi) \]

Optimization for \( \mu_1, \mu_2 \) is trivial.
Alternating minimization.
Shape regularization

- Obviously, some sort of regularization is needed for the shape of the contour.
  - For example, without the curve length term, the Chan-Vese model simplifies into the *k-means* algorithm.
- A natural choice of regularization for curves is to use some measure of the curve:
  - Often, the length of the contour is added to the functional, resulting in the addition of a curvature flow term.
  - Another possibility is affine curvature.
  - The geodesic active contours model contains its own shape regularization.
- However, the silhouette or boundary of most objects we try to segment should be based on more than Occam’s razor.
Linear Shape Spaces

- Look at the linear space of signed distance maps of a given shape, based on several examples.
- Perform PCA for these SDM's (requires alignment).

Figure: Signed distance maps of corpus callosum examples. Leventon et. al. '00
Linear Shape Spaces (cont.)

- Obtain a statistical representation of common variations in the object shape.

Figure: First 3 modes of corpus callosum examples. Leventon et. al. ’00
Linear Shape Spaces (cont.)

- Given the current levelset, its representation in the linear shape space is given by
  \[ \alpha = U_k^T (u - \mu), \]
- where \( U_k \) is the matrix of vectors (read: SDMs) spanning the examples of SDMs, obtained by PCA.
- or, as an SDM,

\[ \tilde{u} = U_k \alpha \]

All of the above discussion assumes that the contour is registered using the pose parameter \( p \).
Linear Shape Spaces (cont.)

- Given a current contour, its “correct” representation is given by a maximum a-posteriori (MAP) estimation.

\[
P(\alpha, p|u, I) = \frac{P(u, I|\alpha, p)P(\alpha, p)}{P(u, I)} = \frac{P(u|\alpha, p)P(I|\alpha, p, u)P(\alpha)P(p)}{P(u, I)}.
\]

- As often happens, \(\alpha\) is assumed to be Gaussian.

\[\alpha \sim N(0, \Sigma_\alpha)\]

- where \(\Sigma_\alpha\) is computed from the given examples.

- \(u\) is connected to \(\alpha, p\) using \(\tilde{u}\).

- \(I\) is only statistically linked to \(u\) in the model,

\[
P(I|\alpha, p, u) = P(I|u)
\]
Non-Linear Shape Spaces

Later works have extended the model:

▶ Chen et. al. ’01, Tsai et. al. ’01, Rousson and Paragios ’02,
▶ Cremers ’02,
▶ Damberville ’06, Yezzi et. al. ’07,
▶ NLDR Algorithms in use: KPCA, LLE, Diffusion maps - and more..
▶ Extensions by different solutions to the pre-image problem, incorporation of dynamics, different assumption on the transformations allowed and more..,
Projective Geometry Shape Priors

- Damberville, Sandhu ’08.
- As always in computer vision - if your model is accurate, you can expect much better results.
- Shape spaces work fine in medical images, where you really have complete information of the object (once reconstructed..)
- Doesn’t work as well for 3D object and a single viewpoint.
- Different object poses are not approximated well by a linear space, or by close-to-linear spaces.
- Instead, why not parameterize the contour as an object silhouette?
- If the shape is variable, the variations should be in terms of the 3D object, not its projected silhouette.
Projective Geometry Shape Priors (cont.)

**Figure:** Taken from Sandhu et. al., '09: Initial pose and shape, AC result, result using prior.