Exercise 6.1

1. Let $N$ be a set of $n \geq 2$ voters and $M$ be a set of $m \geq 3$ alternatives. The Borda rule assigns to any voter’s ranked alternatives the scores $0, 1, \ldots, m - 1$, respectively (so the first in the ranking gets 0 points, the second gets 1 point, etc.), and for any profile of voters’ rankings, choose a social ranking according to the sum of scores, where the lower the sum the earlier (i.e. better) the alternative appears in the social ranking. Assume some lexicographic tie-breaking rule.

   Prove or disprove: the Borda rule satisfies Pareto.

   Prove or disprove: the Borda rule satisfies IIA.

   Prove or disprove: Truth-telling is a dominant strategy under the Borda rule.

2. Let $N$ be a set of $n \geq 2$ voters and $M$ be a set of $m \geq 3$ alternatives. A voting rule is called unanimous if whenever the most preferred alternative of all voters is the same, then this alternative is chosen first in the social ranking. Prove: there is a no non-dictatorial unanimous voting rule that satisfies IIA. Prove or disprove: there is a non-dictatorial unanimous voting rule.