Exercise 3

1. Find the optimal (revenue maximizing) reserve price for 2 person second price auction with valuations independently and identically distributed uniformly on \([0, 1]\).

2. Assume the valuation of \(n\) bidders are \(v_1 \geq v_2 \ldots \geq v_n \geq 0\) and that these values are common knowledge to all bidders. In particular, every bidder knows not only his value but the values of all other bidders as well. Prove that in general there is no pure strategy equilibrium in first price auction. Assume that all valuations are integers, and all bids are allowed to be only integers. Can you find an equilibrium to 1st price auction?

3. Given a game in strategic form, a strategy profile \(a = (a_1, \ldots, a_n) \in A\) is dominated for \(i\) if there is \(b \in A_i\), such that \(u_i(b, c_{-i}) \geq u_i(a_i, c_{-i})\) for every \(c_{-i} \in A_{-i}\) with at least one strict inequality. We will say an equilibrium profile is sensible if it is not dominated for any agent.

A 3rd price auction is an auction where the highest bidder win and pays the 3rd highest price (we assume at least 3 bidders). Prove that 3rd price auction with complete information and discrete valuations/ bids possesses a sensible equilibrium that obtains the highest valuation as its revenue. Can this be obtained in 1st or 2nd price auction?

4. Consider a setting with a single good \(g\) and \(n \geq 2\) bidders with private valuations for \(g\) selected uniformly and identically from the interval \([0, 1]\). Design an auction that in equilibrium for any odd bidder the fact he is winner/loser as well as his payment will be as in equilibrium if all participants would have participated in first-price auction (for any profile of valuations), and for any even bidder the fact he is winner/loser and his payment will be as in equilibrium if all participants would have participated in second-price auction (for any profile of valuations).
Specify in detail the allocation rule and the payment rule of the suggested auction; then specify the equilibrium behavior and prove that it is an equilibrium, and that it satisfies the above desired requirements.

5. Exercises from book (not for delivery): chapter 9 (1,2,3,4,5,6,7,8,9,10,11).