Exercise 2

1. Given a game $G$, with a finite set of players $N$, and a finite set of actions $A_i$ for each player, a profile of actions $a \in A = \Pi_{i=1}^n A_i$ is called a strong equilibrium if $\forall B \subseteq N$, and $\forall s_B \in \Pi_{j \in B} A_i$, $\exists l \in B$ such that $u_i(a_{-B}, s_B) \leq u_i(a)$.

A network traffic game is a parallel-links congestion game, if in the graph $G = (V, E)$, $E = \bigcup_j P_j$ where any $P_j$ is a path from $s$ to $t$, and $P_p \cap P_q = \emptyset$ for any $p \neq q$.

Prove: all pure Nash equilibria of a parallel links game are strong equilibria.

2. A (generalized) traffic network is a DAG $G = (V, E)$, with two distinguished set of nodes $V_s$ and $V_t$ where there is path from any $v_s \in V_s$ to any one of the nodes $v_t \in V_t$. Each $e \in E$ is associated with a cost function $c_e : Z^+ \rightarrow \mathbb{R}^+$ where $c_e(0) = 0$. Let $P_{v_s}$ be the set of (simple) paths from $v_s$ to any of nodes $v_t \in V_t$, where for every $p = (v_s, v_1, v_2, \ldots, v_k, v_t) \in P_{v_s}$ we define $c(p) = c_{(v_s, v_1)} + c_{(v_1, v_2)} + c_{(v_2, v_3)} + \cdots + c_{(v_{k-1}, v_k)} + c_{(v_k, v_t)}$.

Given a set of players $N = \{1, 2, \ldots n\}$, and a (generalized) traffic network, and a mapping $r : N \rightarrow V_s$, the corresponding congestion game is defined by having the set of strategies of each agent $i$ be $P_{r(i)}$, and for $p = (p_1, \ldots p_n)$, where $l_e(p) = |\{i : p_i \text{ includes edge } e\}|$ we have

$$u_i(p) = \sum_{e \in p} c_e(l_e(p))$$

Prove or disprove: every (generalized) network traffic game, possess a pure strategy equilibrium.

3. Exercises from book (not for delivery): chapter 8 (1,2,3,4)