Assignment 4
Due Jan 12, 2017

Part 1: Counting Repairs

In class, we have seen that for the schema $S$ that consists of the relation $R(A, B)$ with the FDs $A \rightarrow B$ and $B \rightarrow A$, the problem of counting the repairs of an inconsistent database is the same as the problem $\#\text{MaxMatch}$: count the maximal matchings in a bipartite graph. Consequently, we concluded that counting the repairs for $S$ is $\#\text{P}$-hard (and in fact, $\#\text{P}$-complete).

**Question 1.1.** Prove that $\#\text{MaxMatch}$ efficiently reduces to counting the repairs of the schema that consists of the ternary relation $S(A, B, C)$ with the FDs $A \rightarrow B$ and $B \rightarrow C$.

**Question 1.2.** Answer the same question as 1.1, but now with the FDs $A \rightarrow B$ and $C \rightarrow B$.

Part 2: Consistent Query Answering

Consider the Boolean CQ $Q() := R(\underline{x}, y), S(y, x, z), T(z)$.

**Question 2.1.** Using the trichotomy theorem of Koutris and Wijsen (studied in class), show that $\text{Consistent}_{\Sigma}^Q$ cannot be phrased in First-Order Logic.

**Question 2.2.** Devise a polynomial-time algorithm for computing $\text{Consistent}_{\Sigma}^Q(I)$ on a given input instance $I$. (Hint: adapt the strategy for $R(\underline{x}, y), S(y, x)$ we saw in class.)

Part 3: Aggregate on Repairs

In some historical town, the mayor was replaced every year, and a mayor could not serve more than one term in a lifetime. An inconsistent database describes the recorded mayors in the relation $\text{Mayors}(\text{name}, \text{year})$ with the FDs $\text{name} \rightarrow \text{year}$ and $\text{year} \rightarrow \text{name}$. The following SQL query $Q$ asks for the latest year for which we have a record.

\[
\text{SELECT MAX(year) FROM Mayors}
\]

Given an inconsistent instance $I$, we are interested in the extremal (maximal and minimal) values:

\[
Q_{\text{max}}^\text{max}(I) \overset{\text{def}}{=} \max \{Q(J) \mid J \in \text{Repairs}_{\Sigma}(I)\}
\]

\[
Q_{\text{min}}^\text{min}(I) \overset{\text{def}}{=} \min \{Q(J) \mid J \in \text{Repairs}_{\Sigma}(I)\}
\]

Devise polynomial-time algorithms for computing $Q_{\text{max}}^\text{max}(I)$ and $Q_{\text{min}}^\text{min}(I)$ on the given $I$. (Hint: the former is easy, the latter is more intricate.)

Good luck!