1. The *Immediate snapshot* complex, $\mathcal{IS}(\sigma)$ is defined as follows (see Page 21 in the “manifolds” notes). Each vertex in this complex has the form $(P_i, \sigma_i)$, where $P_i$ is the process taking the steps, and $\sigma_i$, the result of its snapshot, is a face of the input simplex $\sigma$. Each simplex in the protocol complex satisfies the following properties.

- **Property 3.2.1.** Each process write appears in its own view.
- **Property 3.2.2.** Snapshots are ordered.
- **Property 3.2.3.** Each snapshot is ordered immediately after its write.

Provide a formula for the number of simplexes in $\mathcal{IS}(\sigma^n)$.

2. The *Snapshot* complex, $\mathcal{S}(\sigma)$ has a similar structure to the immediate snapshot complex, but the simplexes only have to satisfy Properties 3.2.1 and 3.2.2.

This complex is achieved when each process writes its value, then takes an atomic snapshot and halts with this snapshot as its output value.

- Prove that any face of $\mathcal{S}(\sigma^n)$ is contained in at most $n$ simplexes.
- Is $\mathcal{S}(\sigma^1)$ (the snapshot complex for two processes) a manifold?
- Is $\mathcal{S}(\sigma^2)$ (the snapshot complex for three processes) a manifold?
- Draw $\mathcal{S}(\sigma^2)$.

3. Use reasoning similar to the proof of Lemma 3.4.1, to prove the *bridges of Königsberg* problem:

The city of Königsberg was set on both sides of a River, and included two large islands which were connected to each other and the mainland by seven bridges. The problem is to find a walk through the city that would cross each bridge once and only once. The islands could not be reached by any route other than the bridges, and every bridge must have been crossed completely every time (one could not walk half way onto the bridge and then turn around and later cross the other half from the other side). (From Wikipedia.)