Mathematical Techniques in the Theory of Distributed Computing, 236602
Spring 2013, Home Assignment 4

1. Let $G$ be a path of $n$ nodes $v_1, \ldots, v_n$, i.e., the set of edges is $\{\{v_i, v_{i+1}\} \mid 1 \leq i \leq n - 1\}$. Assume that the node $v_1$ initially holds a rumor. Prove a tight bound for the number of rounds required for rumor spreading. That is, prove a lower bound on this complexity, and show an algorithm that attains this bound.

2. Let $G$ be a star graph of $n$ nodes $v_1, \ldots, v_n$, i.e., the set of edges is $\{\{v_n, v_i\} \mid 1 \leq i \leq n - 1\}$. Assume that the center node $v_n$ initially holds a rumor, and consider running randomized rumor spreading with a uniform distribution under the $\text{PUSH}$ model. Analyze the number of rounds it takes for all nodes to have the rumor with high probability.

3. This question discusses expansion of $d$-regular graphs, which are graphs where every node has a constant degree $d$. The vertex-expansion of a graph $G$ is
   \[ \alpha(G) = \min_{S \subseteq V, |S| \leq n/2} \frac{|N(S)|}{|S|}, \]
   where $N(S)$ is the set of vertices in $V \setminus S$ which have at least one neighbor in $S$. The edge-expansion of $G$ is
   \[ h(G) = \min_{S \subseteq V, |S| \leq n/2} \frac{|E(S, V \setminus S)|}{|S|}, \]
   where $E(S, V \setminus S)$ is the set of edges with exactly one endpoint in $S$.

   (a) Prove that if $G$ is a $d$-regular graph then its edge-expansion and vertex-expansion are in the same order. That is, prove that $\alpha(G) = \Theta(h(G))$.

   (b) Show an example of a $d$-regular graph with expansion $O(\frac{1}{n^{1/d}})$. (Note that a random $d$-regular graph has a constant expansion with high probability.)

Submission date: 20/6/2013.

Try to solve the problems by yourself, and in any case write the solution by yourself.

For each question please write if you got help, from whom, and how much.