1. Prove that in a connected (1-interval connected) dynamic network of size $n$, the counting problem can be solved in $O(n)$ rounds using messages of size $O(n \log n)$. Find the smallest number of rounds that you can for the $O(n)$ expression.

2. For the $T$-token dissemination algorithm we saw in class for $2T$-interval connected graphs, prove the claim that for every node $u$, token $t$, and round $r$ such that $t\text{dist}_i(u, t) \leq r$, one of the following holds:
   
   (a) $t \in S_{u,i,r+1}$, or
   
   (b) $S_{u,i,r+1}$ contains at least $r - t\text{dist}_i(u, t)$ tokens that are smaller than $t$.

3. A graph $G = (V, E)$ is called $k$-vertex connected if for very two nodes $u, v \in V$, there are $k$ vertex-disjoint paths connecting $u$ and $v$. Show that if the dynamic graph $\{G_i\}_{i=1}^{\infty}$ is always $k$-vertex connected (that is, $G_i$ is $k$-vertex connected for all $i \geq 1$), then the 1-token dissemination problem can be solved in $O(n/k)$ rounds.

4. Consider the dynamic network setting where the size of messages is bounded by the size of a single token, and consider the problem of disseminating $n$ tokens that are all given as input to the same node $v$. A deterministic algorithm is knowledge-based if the token a node sends in a round is a function of its ID, the round number, and the set of tokens it knows. Prove that in the above setting, any knowledge-based algorithm requires $\Omega(n^2)$ rounds to complete on a 1-interval connected graph.

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Try to solve the problems by yourself, and in any case write the solution by yourself.

For each question please write if you got help, from whom, and how much.