1. Warm-up:
   Let $G$ be a graph with a $k_1$-spanner $G_1$. Let $G_2$ be a $k_2$-spanner of $G_1$. For which $k$ is $G_2$ a $k$-spanner of $G$? Prove your answer.

2. The girth conjecture:
   Erdos conjectured that there exist graphs with $n$ vertices that have $\Omega(n^{1+1/k})$ edges and girth $2k + 1$, where girth is the length of the shortest cycle. Show that if this is true, then there are graphs for which any $(2k - 1)$-spanner has at least $\Omega(n^{1+1/k})$ edges.

3. Spanning bipartite graphs:
   (a) Show that a bipartite graph has no $2$-spanner except for itself.
   (b) Let $G$ be a bipartite graph that has no $(2k - 1)$-spanner except for itself. Show that $G$ also does not have a $2k$-spanner except for itself.

4. Relations between synchronizers and spanners:
   (a) Prove that if $G$ has a $t$-spanner with $m$ edges then it has a synchronizer $s$ with $C = O(tm)$ and $T = O(t)$.
   (b) Prove that if $G$ does not have a $t$-spanner with at most $m$ edges then every synchronizer has either $C \geq m + 1$ or $T \geq t + 1$.

Submission date: 2/5/2013.

Try to solve the problems by yourself, and in any case write the solution by yourself.

For each question please write if you got help, from whom, and how much.
Questions marked by (*) are harder. Please write a solution only if you solved it on your own.