A. Distributed algorithms

Example 1: synchrony

Exercise 1: find a trade-off between no. of days and no. of presidents.

Example 2: leader election

message passing
asynchronous

Exercise 2: find a better algorithm to find the maximum, prove correctness and analyze performance.
Example 3: faults

Impossibility of consensus

The Byzantine Generals Problem

Example 4: snapshot
B. Self Stabilization

Example 5: Unison (stabilizing clocks)

Program at each processor $p$:
choose a neighbor $x$ of $p$;
if $t(p) = t(x)$ then $t(p) := t(p) + 1$;

Stabilizing if all start with the same time, however …

Is it stabilizing? no! (may even deadlock)
Program at each processor $p$:

choose a neighbor $x$ of $p$:

if $t(p) = t(x)$ then $t(p) := t(p)+1$;
   
   if $t(p) < t(x)$ then $t(p) := t(x)$;

Is it stabilizing?
Program at each processor $p$:

choose $c$ fairly; or $x$ of $p$

if $t(p) = t(x)$ then $t(p) := t(p) + 1$;
if $t(p) < t(x)$ then $t(p) := t(x)$;

Is it stabilizing? yes!

Exercise 3: prove and analyze time until stabilization.

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**C. Optical networks**

- **lightpaths**
- **Valid coloring**

$w(p_1) \neq w(p_2)$

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Saving a switch:
Example 6: min ADMs

Exercise: prove NP=hardness

Example 7: min ADMs w/grooming

g=2

15
Input: a graph and a set of paths
Output:
• a valid coloring
• a valid coloring with minimum number of ADMs (number of colors can be a parameter)
  • a valid coloring with minimum number of ADMs, given a grooming factor (number of colors can be a parameter)

Problems

Objective function:
Minimize the number of ADMs

Cases:
  w/out grooming (g = 1)
  w/ grooming (g > 1)

Questions:
  Complexity?
  Approximability?

Example 8: approximation algorithms

Objective function:
Minimize the number of ADMs

Cases:
  w/out grooming (g = 1)
  w/ grooming (g > 1)

Questions:
  Complexity?
  Approximability?

N: # of paths.
ALG: # of ADMs used by the algorithm.
OPT: # of ADMs used by an optimal solution.
ALG ≤ 2N
N ≤ OPT
ALG/OPT ≤ 2N/N=2
   with grooming:
      ALG/OPT ≤ 2 g
Example 9: min regenerators

\[ d=2, \quad g=1 \]

set \( A = 3 \) \quad set \( B = 1 \)

set \( A \) and set \( B = 4 \)
Input arrives one at a time, and a decision is made (and cannot be changed).

In the minADM problem: lightpaths arrive one at a time, and need to be colored.

Competitive analysis

An on-line algorithm $A$ is $c$-competitive if $A(I) \leq c \cdot OPT(I)$ for any input sequence $I$. ($A(I)$ and $OPT(I)$ are #ADMs used by $A$ and by an optimal offline algorithm $OPT$.)

Example 10: on-line algorithms

Case a: $\frac{7}{4} = 1.75$

Any algorithm $\geq \frac{7}{4}$, even for a ring
Case b:
Case b1: 6/3 = 2
Case b2: 5/3 = 1.67

any algorithm ≥ 1.67

Exercise: prove any algorithm ≥ 1.75

any algorithm for a path ≥ 3/2

k paths

k-1 spaces:
x between same color
k-1-x between different colors

So far: any algorithm uses 2k ADMs

now - a short path at each gap of different color
k=12, x=6, 12-1-6=5

Any algorithm uses at least one more ADM for each (ALG uses exactly one)

So: any algorithm ≥ 2k + (k-1-x) ADMs
So far: use $\geq 2k \cdot (k-1-x)$ ADMS

now - two long paths at each of the $k$ gap of same color

Any algorithm must use 2 ADMs for each

So: any algorithm $\geq 2k \cdot (k-1-x) + 4x = 3k + 3x - 1$ ADMs

We showed: any algorithm uses $\geq 3k + 3x - 1$ ADMs

2 k ADMS

OPT: the short paths $\leq 2k$ ADMs

for the long paths $\leq 2x$ ADMs

OPT $\leq 2k + 2x$

any algorithm/OPT $\geq 3/2 - 1/(2k)$

D. ATM networks

Virtual path

Virtual channel

load = 3

(same)

hop count = 3 (space)

stretch factor = 4/3
Given a network, find an “Optimal Layout” such that:

(A) Given an upper bound on the hop count, minimize the load.

N=7, Max hop count 1 => Min load 6

(B) Given an upper bound on the load, minimize the hop count.

N=7, Max load 1 => Min hop count 6

l = 3    h = 2    f(3,2) = 10
Exercise 4: how many points can you connect, given $l$ and $h$?
Example 12: Decision problems

**Input:** Graph $G$, a vertex $v$, $h$, $\ell > 0$.

**Question:** is there a VP layout for $G$, by which $v$ can reach all other nodes, with hop count bounded by $h$ and load bounded by $\ell$?