A. Distributed algorithms

Example 1: synchrony

Exercise 1: find a trade-off between no. of days and no. of presidents.
Example 2: leader election

message passing
asynchronous

Exercise 2: find a better algorithm to find the maximum, prove correctness and analyze performance.
Example 3: faults

Impossibility of consensus

The Byzantine Generals Problem
Example 4: snapshot
Example 5: Unison (stabilizing clocks)

clock synchronization
shared memory, synchronous
Program at each processor $p$:

choose a neighbor $x$ of $p$;

if $t(p) = t(x)$ then $t(p) := t(p)+1$;

Stabilizing if all start with the same time, however ...
Is it stabilizing? no! (may even deadlock)

\text{if } t(p) = t(x) \text{ then } t(p) := t(p)+1;
Program at each processor $p$:

choose a neighbor $x$ of $p$;

if $t(p) = t(x)$ then $t(p) := t(p) + 1$;

if $t(p) < t(x)$ then $t(p) := t(x)$;

Is it stabilizing?
if $t(p) = t(x)$ then $t(p) := t(p) + 1$;
if $t(p) < t(x)$ then $t(p) := t(x)$;

Is it stabilizing? yes!
Is it stabilizing? no!

if \( t(p) = t(x) \) then \( t(p) := t(p) + 1; \)
if \( t(p) < t(x) \) then \( t(p) := t(x); \)
Program at each processor $p$:

choose a neighbor $x$ of $p$ fairly;

if $t(p) = t(x)$ then $t(p) := t(p)+1$;
if $t(p) < t(x)$ then $t(p) := t(x)$;

Is it stabilizing? yes!

Exercise 3: prove and analyze time until stabilization.
C. Optical networks

Valid coloring

\[ w(p_1) \neq w(p_2) \]
Saving a switch:
Example 6: min ADMs

Exercise: prove NP=hardness
Example 7: min ADMs w/grooming

colors=3
switches=10

g=2

colors=2
switches=9
Input: a graph and a set of paths

Output:

• a valid coloring

• a valid coloring with minimum number of ADMs (number of colors can be a parameter)

• a valid coloring with minimum number of ADMs, given a grooming factor (number of colors can be a parameter)
Objective function: Minimize the number of ADMs

Cases:
- w/out grooming (g = 1)
- w/ grooming (g > 1)

Questions:
- Complexity ?
- Approximability ?
$N$: # of paths.
$ALG$: # of ADMs used by the algorithm.
$OPT$: # of ADMs used by an optimal solution.
$ALG \leq 2N$
\[ N \leq OPT \]
$ALG/OPT \leq 2N/N=2$

with grooming:
$ALG/OPT \leq 2g$
Example 9: min regenerators
set $A = 3$  \hspace{1cm} set $B = 1$  

d = 2  

g = 1
set A and set B = 4

d=2

g=1
set A or set B = 3
Example 10: on-line algorithms

- Input arrives one at a time, and a decision is made (and cannot be changed).
- In the minADM problem: lightpaths arrive one at a time, and need to be colored.

Competitive analysis

- An on-line algorithm $A$ is $c$-competitive if

$$A(I) \leq c \cdot OPT(I)$$

for any input sequence $I$. 

($A(I)$ and $OPT(I)$ are #ADMs used by $A$ and by an optimal offline algorithm $OPT$.)
Case a: \[ \frac{7}{4} = 1.75 \] Any algorithm \( \geq \frac{7}{4} \), even for a ring
Case b:
- Case b1: \( \frac{6}{3} = 2 \)
- Case b2: \( \frac{5}{3} = 1.67 \)

Any algorithm \( \geq 1.67 \)

Exercise: prove any algorithm \( \geq 1.75 \)
any algorithm for a path $\geq 3/2$

$k$ paths

$k-1$ spaces:

$x$ between same color

$k-1-x$ between different colors

$k=12$

$x=6$
So far: any algorithm uses $2k$ ADMs

now - a short path at each gap of different colors

Any algorithm uses at least one more ADM for each (ALG uses exactly one)

So: any algorithm $\geq 2k + (k-1-x)$ ADMs

$k=12$, $x=6$, $12-1-6=5$
So far: use $\geq 2k + (k-1-x)$ ADMs

now - two long paths at each of the $k$ gap of same color

Any algorithm must use 2 ADMs for each

So: any algorithm $\geq 2k + (k-1-x) + 4x = 3k+3x-1$ ADMs
We showed: any algorithm uses \( \geq 3k + 3x - 1 \) ADMs

2k ADMs OPT: the short paths \( \leq 2x \) ADMs for the long paths

\( \text{OPT} \leq 2k + 2x \)

any algorithm/OPT \( \geq 3/2 - 1/(2k) \)
D. ATM networks

load = 3 (space)

hop count = (time) stretch factor = 4/3
Example 11: Optimal designs

Given a network, find an “Optimal Layout” such that:

(A) Given an upper bound on the hop count, minimize the load.

$N=7, \text{Max hop count} 1 \Rightarrow \text{Min load} 6$
Given an upper bound on the load, minimize the hop count.

N=7, Max load 1 => Min hop count 6
\( l = 3 \quad h = 2 \quad f(3,2) = 10 \)
$l = 2 \quad h = 3 \quad f(2,3) = 10$
Exercise 4: how many points can you connect, given l and h?
כדור 2-ממדי ברדיוס 3:
כדור 2-ממדי ברדיוס 1

כדור 1-ממדי ברדיוס 2:
Input: Graph $G$, a vertex $v$, $h$, $l > 0$.
Question: is there a VP layout for $G$, by which $v$ can reach all other nodes, with hop count bounded by $h$ and load bounded by $l$?
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<th>2</th>
<th>3</th>
<th>...</th>
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<td>P</td>
<td>P</td>
<td>...</td>
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<tr>
<td>2</td>
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