Subdivision Surfaces
Geometric Modeling

• Sometimes need more than polygon meshes
  – Smooth surfaces

• Traditional geometric modeling used NURBS
  – Non uniform rational B-Spline
  – Demo
Problems with NURBS

• A single NURBS patch is either a topological disk, a tube or a torus

• Must use many NURBS patches to model complex geometry

• When deforming a surface made of NURBS patches, cracks arise at the seams
Subdivision

“Subdivision defines a smooth curve or surface as the limit of a sequence of successive refinements”
Subdivision Surfaces

• Generalization of spline curves / surfaces
  – Arbitrary control meshes
  – Successive refinement (subdivision)
  – Converges to smooth limit surface
  – Connection between splines and meshes
Subdivision Surfaces

- Generalization of spline curves / surfaces
  - Arbitrary control meshes
  - Successive refinement (subdivision)
  - Converges to smooth limit surface
  - Connection between splines and meshes
Example: Geri’s Game (Pixar)

• Subdivision used for
  – Geri’s hands and head
  – Clothing
  – Tie and shoes
Example: Geri’s Game (Pixar)

Woody’s hand (NURBS)  Geri’s hand (subdivision)
Example: Geri’s Game (Pixar)

• Sharp and semi-sharp features
Example: Games

Supported in hardware in DirectX 11
Subdivision Curves

Given a control polygon...

...find a smooth curve related to that polygon.
Subdivision Curve Types

- Approximating
- Interpolating
- Corner Cutting
Approximating
Approximating

Splitting step: split each edge in two
Approximating

Averaging step: relocate each (original) vertex according to some (simple) rule...
Approximating

Start over ...
Approximating

...splitting...
Approximating

...averaging...
Approximating

...and so on...
Approximating

If the rule is designed carefully...

... the control polygons will converge to a smooth limit curve!
Equivalent to …

• Insert *single* new point at mid-edge

• *Filter* entire set of points.

Catmull-Clark rule: Filter with \((1/8, 6/8, 1/8)\)
Corner Cutting

• Subdivision rule:
  – Insert *two* new vertices at $\frac{1}{4}$ and $\frac{3}{4}$ of each edge
  – *Remove* the old vertices
  – Connect the new vertices
Interpolating (4-point Scheme)

- Keep old vertices
- Generate new vertices by fitting cubic curve to old vertices
- $C^1$ continuous limit curve

\[ f(x) = ax^3 + bx^2 + cx + d \]
\[ f(j) = p_{i+j} , \quad j = 0, \ldots, 3 \]
\[ q_i = f(3/2) \]
\[ = \frac{1}{16} (-p_i + 9p_{i+1} + 9p_{i+2} - p_{i+3}) \]
Interpolating
Interpolating
Interpolating
Interpolating
Interpolating
Subdivision Surfaces

• No regular structure as for curves
  – Arbitrary number of edge-neighbors
  – Different subdivision rules for each valence
Subdivision Rules

- How the connectivity changes

- How the geometry changes
  - Old points
  - New points
Subdivision Zoo

• Classification of subdivision schemes

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td><strong>Primal</strong></td>
<td>Faces are split into sub-faces</td>
</tr>
<tr>
<td><strong>Dual</strong></td>
<td>Vertices are split into multiple vertices</td>
</tr>
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</table>

| **Approximating** | Control points are not interpolated |
| **Interpolating** | Control points are interpolated               |
## Subdivision Zoo

- Classification of subdivision schemes

<table>
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<tr>
<th>Primal (face split)</th>
<th>Triangular meshes</th>
<th>Quad Meshes</th>
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<tr>
<td><strong>Approximating</strong></td>
<td>Loop ($C^2$)</td>
<td>Catmull-Clark ($C^2$)</td>
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<tr>
<td><strong>Interpolating</strong></td>
<td>Mod. Butterfly ($C^1$)</td>
<td>Kobbelt ($C^1$)</td>
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- Many more...
## Subdivision Zoo

- Classification of subdivision schemes

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Catmull-Clark Subdivision

- Generalization of bi-cubic B-Splines
- Primal, approximation subdivision scheme
- Applied to polygonal meshes
- Generates $G^2$ continuous limit surfaces:
  - $C^1$ for the set of finite extraordinary points
    - Vertices with valence $\neq 4$
  - $C^2$ continuous everywhere else
Catmull-Clark Subdivision

\[ V_2 = \frac{1}{n} \times \sum_{j=1}^{n} d_j \]

\[ E_i = \frac{1}{4} \left( d_1 + d_{2i} + V_i + V_{i+1} \right) \]

\[ d'_1 = \frac{(n-3)}{n} d_1 + \frac{2}{n} R + \frac{1}{n} S \]

\[ R = \frac{1}{m} \sum_{i=1}^{m} E_i \quad S = \frac{1}{m} \sum_{i=1}^{m} V_i \]
Catmull-Clark Subdivision
## Classic Subdivision Operators

- Classification of subdivision schemes

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- **Primal Operators**
  - **Triangles**
    - Loop
  - **Rectangles**
    - Catmull-Clark

- **Interpolating Operators**
  - Butterfly
  - Kobbelt

- **Dual Operators**
  - Doo-Sabin Midedge
Loop Subdivision

• Generalization of box splines
• Primal, approximating subdivision scheme
• Applied to triangle meshes
• Generates $G^2$ continuous limit surfaces:
  – $C^1$ for the set of finite extraordinary points
    • Vertices with valence $\neq 6$
  – $C^2$ continuous everywhere else
Loop Subdivision

\[ E_i = \frac{3}{8} (d_1 + d_i) + \frac{1}{8} (d_{i-1} + d_{i+1}) \]

\[ d'_1 = \alpha_n d_1 + \frac{(1 - \alpha_n)^{n+1}}{n} \sum_{j=2}^{n} d_j \]

\[ \alpha_n = \frac{3}{8} + \left( \frac{3}{8} + \frac{1}{4} \cos \frac{2\pi}{n} \right)^2 \]
Loop Subdivision
# Subdivision Zoo

- Classification of subdivision schemes

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Doo-Sabin Subdivision

- Generalization of *bi-quadratic B-Splines*
- Dual, approximating subdivision scheme
- Applied to *polygonal* meshes
- Generates \( G^1 \) *continuous* limit surfaces:
  - \( C^0 \) for the set of finite extraordinary points
    - Center of irregular polygons after 1 subdivision step
  - \( C^1 \) continuous everywhere else
Doo-Sabin Subdivision

\[
V_2 = \frac{1}{n} \times \sum_{j=1}^{n} d_j
\]

\[
E_i = \frac{1}{2} \left( d_1 + d_{2i} \right)
\]

\[
d'_{1,j} = \frac{1}{4} \left( d_1 + E_j + E_{j-1} + V_j \right)
\]
Doo-Sabin Subdivision
# Classic Subdivision Operators

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Butterfly Subdivision

• Primal, interpolating scheme
• Applied to triangle meshes
• Generates $G^1$ continuous limit surfaces:
  – $C^0$ for the set of finite extraordinary points
    • Vertices of valence = 3 or > 7
  – $C^1$ continuous everywhere else
Butterfly Subdivision

\[ E_1 = \frac{1}{2} (d_1 + d_2) + \omega (d_3 + d_4) - \frac{\omega}{2} (d_5 + d_6 + d_7 + d_8) \]

\[ d'_i = d_i \]
Butterfly Subdivision
Remark

• Different masks apply on the boundary

• Example: Loop

\[ a. \text{ Masks for odd vertices} \quad b. \text{ Masks for even vertices} \]
Comparison

Doo-Sabin  Catmull-Clark
Loop  Butterfly
Comparison

• Subdividing a cube
  – Loop result is assymetric, because cube was triangulated first
  – Both Loop and Catmull-Clark are better then Butterfly ($C^2$ vs. $C^1$)
  – Interpolation vs. smoothness

Loop

Butterfly

Catmull-Clark
Comparison

- Subdividing a tetrahedron
  - Same insights
  - Severe shrinking for approximating schemes
Comparison

• Spot the difference?
• For smooth meshes with uniform triangle size, different schemes provide very similar results
• Beware of interpolating schemes for control polygons with sharp features
So Who Wins?

- Loop and Catmull-Clark best when interpolation is not required
- Loop best for triangular meshes
- Catmull-Clark best for quad meshes
  - Don’t triangulate and then use Catmull-Clark
Properties of Subdivision

• Flexible modeling
  – Handle surfaces of arbitrary topology
  – Provably smooth limit surfaces
  – Intuitive control point interaction

• Scalability
  – Level-of-detail rendering
  – Adaptive approximation

• Usability
  – Compact representation
  – Simple and efficient code
Beyond Subdivision Surfaces

- Non-linear subdivision [Schaefer et al. 2008]
  Idea: replace arithmetic mean with other function

\[
de\text{Casteljau with }\frac{a+b}{2}
\]
\[
de\text{Casteljau with }\sqrt{ab}
\]
Beyond Subdivision Surfaces

• **T-Splines** [Sederberg et al. 2003]
  – Allows control points to be *T-junctions*
  – Can use less control points
  – Can model different topologies with single surface
Beyond Subdivision Surfaces

• How do you subdivide a teapot?