Signal and Image Processing (236327)

Tutorial 6

Transform Coding
Representation of 1D Discrete Signals

$x$ is a one-dimensional signal of length $N$ (formed as a column).

$U$ is an $N \times N$ unitary matrix, formed of the columns $\{u_i\}_{i=1}^N$.

The transform-domain representation of $x$ is the $N$-length vector $v$ obtained by:

$$v = U^T x$$

$v$ can also be expressed as

$$v = \begin{bmatrix} u_1^T x \\ \vdots \\ u_N^T x \end{bmatrix}$$

i.e., the $i^{th}$ component of $v$ is $v_i = u_i^T x$. 
Representation of 1D Discrete Signals

Given the transform-domain representation $v$, we can return to the signal domain by

$$\hat{x} = Uv$$

For $v = U^T x$ we get

$$\hat{x} = UU^T x = x$$

because the matrix $U$ is unitary, and since we did not processed the representation $v$. Also note that

$$\hat{x} = Uv = \sum_{i=1}^{N} v_i u_i = \sum_{i=1}^{N} (u_i^T x) u_i$$
Transform Coding of 1D Discrete Signals

We can compress the signal via quantization in the transform domain:

$$v^Q = Q(v) \triangleq \begin{bmatrix} Q_1(v_1) \\ \vdots \\ Q_N(v_N) \end{bmatrix} = \begin{bmatrix} Q_1(u_1^T x) \\ \vdots \\ Q_N(u_N^T x) \end{bmatrix}$$

where $Q(\cdot)$ is a vector quantizer, implemented here by applying the $i^{th}$ scalar quantizer $Q_i(\cdot)$ on the $i^{th}$ transform-coefficient $v_i$.

Then, we can obtain the approximation in the signal domain by

$$\hat{x}^{TC} = Uv^Q$$

that can also be expressed as

$$\hat{x}^{TC} = Uv^Q = \sum_{i=1}^{N} Q_i(v_i) u_i = \sum_{i=1}^{N} Q_i(u_i^T x) u_i$$

In general, $\hat{x}^{TC} \neq x$. 