Complexity of algebraic computation

Homework assignment #7

1. In the Euclidean algorithm from Lecture 4, for the case of integers, prove that $f_k \geq 2f_{k+2}$.

2. Explain why step 2 on page 15 of lecture 7 is required.

3. **Definition** An $n$th root of unity $\omega$ is *principal* if for all $k = 1, 2, \ldots, n - 1$, \[
\sum_{i=0}^{n-1} \omega^{ki} = 0.
\]

   (a) Let $\omega$ be an $n$th principal root of unity. Prove that if $n$ is not a zero divisor, then $\omega$ is also primitive.

   (b) Is the converse of (a) true? Hint: look at the domains with zero divisors.

   (c) Let $n$ be a power of 2 and let $\omega^{n/2} = -1$. Show that $\omega$ is an $n$th a principal root of unity.