Complexity of algebraic computation

Homework assignment #1

1. For a given positive integer \(n\), give an upper bound on the number of algebraic operations needed to compute \(x^n\).

2. Prove that if the product of two \(c \times c\) matrices can be computed in \(a\) multiplications, then \(M(n) = O(n \log_c a)\).

3. Prove that if the product of an \(\ell' \times m'\) matrix by an \(m' \times n'\) matrix can be computed in \(M'\) multiplications and the product of an \(\ell'' \times m''\) matrix by an \(m'' \times n''\) matrix can be computed in \(M''\) multiplications, then the product of an \(\ell' \ell'' \times m'm''\) matrix by an \(m'm'' \times n'n''\) matrix can be computed in \(M'M''\) multiplications.

4. Let \(S(n)\) denote the minimal number of operations (additions and multiplications) needed for computing the square of an \(n \times n\) matrix. Prove that if \(S(n) = O(n^c)\), then \(M(n) = O(n^c)\) as well.