Subdivision Schemes
What is Subdivision?

- Subdivision is a process in which a poly-line/mesh is recursively refined in order to achieve a smooth curve/surface.

- Two main groups of schemes:
  - Approximating - original vertices are moved
  - Interpolating – original vertices are unaffected

Is the scheme used here interpolating or approximating?
Why Subdivision?

Frame from "Geri's Game" by Pixar
Why Subdivision?

- LOD
- Compression
- Smoothing

52Kb → 1Kb

13.3Mb → 424Kb
Corner Cutting
Corner Cutting
Corner Cutting
Corner Cutting
Corner Cutting
Corner Cutting

control point

limit curve

control polygon
This result is a quadratic B-spline curve. Known as Chaikin scheme.
Cubic Corner Cutting
Cubic Corner Cutting
Four-Point Scheme
Four-Point Scheme
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control point

control polygon

limit curve
Subdivision Curves

Non interpolatory subdivision schemes
- Corner Cutting

Interpolatory subdivision schemes
- The 4-point scheme
Basic Concepts of Subdivision

- **Subdivision curve**: generated by repeatedly applying a subdivision *operator* to given polygon
- **Initial polygon**: control polygon
- **Central questions:**
  - **Convergence**: Given a subdivision operator and a control polygon, does the subdivision process converge?
  - **Smoothness**: Does the subdivision converge to a smooth curve?
  - **Singular points**: Does the subdivision result in a finite set of singular points?
  - **Derivatives**: How can one compute derivatives of subdivision curves/surfaces?
Subdivision Surfaces

- At each iteration
  - Refine mesh
  - Increase number of vertices
- Mesh vertices converge to a limit surface
  - After infinite number of subdivision steps
- Every subdivision method has:
  - A method to refine the mesh connectivity
  - Rules to calculate location of new vertices
    - And old if they are effected
- A scheme always consists of 2 main parts:
  - A method to generate the \textit{topology} of the new mesh.
  - Rules to determine the \textit{geometry} of the vertices in the new mesh.
Catmull & Clark Subdivision Scheme

- In each iteration:
  - For each face of the mesh, add a face point.
    - Usually at the centroid of the face.
  - For each edge, add an edge point
    - Average of two neighboring face points
  - Connect all face points to edge points
  - Move original points to new positions using a complicated formula
Triangular Subdivision

- Works only for triangle meshes
- Every triangle replaced by 4 new triangles
- Two kinds of new vertices:
  - Green vertices associated with old edges
  - Yellow vertices associated with old vertices
Loop Subdivision

- New vertex is convex combination of old vertices
- List of weights called subdivision mask or stencil
  - Rule for new yellow vertices
    \((n – \text{vertex valence})\)
  - Rule for new green vertices

\[
\begin{align*}
  w_n &= \frac{64n}{40 - \left(3 + 2 \cos \frac{2\pi}{n}\right)^2} - n
\end{align*}
\]
Example
The Limit Surface

- Limit surfaces of Loop’s subdivision is $C^2$ almost everywhere
- Finite set of singular locations where the surface is $C^1$. 
Butterfly Subdivision

- Interpolatory scheme
- New **yellow** vertices inherit location of old vertices
- New **green** vertices computed by following stencil:

\[
\begin{array}{cccc}
-1 & 8 & 2 & -1 \\
8 & 2 & 8 & -1 \\
2 & 8 & 2 & -1 \\
-1 & -1 & -1 & -1 \\
\end{array}
\]
Example
Limit surfaces of Butterfly subdivision are $C^1$, but do not have second derivative.
Comparison
Properties

- Require regular connectivity (valence 6) to work well
- Easy to implement
- Local support
- Allow LOD
- Continuous

Drawbacks
- Not always intuitive
- Can have artifacts
- Sometimes difficult to control