Revision for The Test

Good Luck!
Question 1: Transformations

1. Write a 3D transformation that reflects a point through a plane \( Ax + By + Cz + D = 0 \), in matrix form, using only the parameters \( (A, B, C, D) \).
Answer 1a:

- Translation by (0,0,-D/C)
  - Plane becomes Ax+BY+CZ=0

- Rotation (representation) by an axes system of two unit vectors on the plane and its normal:
  \[
  \begin{align*}
  \begin{pmatrix}
  -B, A, 0 \\
  0, -C, B \\
  A, B, C
  \end{pmatrix}
  \begin{pmatrix}
  \sqrt{B^2 + A^2} \\
  \sqrt{B^2 + C^2} \\
  \sqrt{A^2 + B^2 + C^2}
  \end{pmatrix}
  \end{align*}
  \]
  - Plane becomes Z=0

- Inversing Z value
- Reverting rotation and transformation
..And in matrix form:

\[(x', y', z', w') = (x, y, z, 1)\]

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & -\frac{D}{C} & 0
\end{pmatrix}
\begin{pmatrix}
-B & 0 & A & 0 \\
\sqrt{A} & -C & B & 0 \\
\sqrt{B} & \sqrt{C} & 0 & 0 \\
0 & \sqrt{D} & \sqrt{E} & \sqrt{F}
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
-B & A & 0 & 0 \\
\sqrt{A} & -C & B & 0 \\
\sqrt{B} & \sqrt{C} & 0 & 0 \\
0 & \sqrt{D} & \sqrt{E} & \sqrt{F}
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 & 0
\end{pmatrix}
\]
Question 1b:

- Does perspective projection preserves Parallel lines?
- If not, where do they meet? What other lines meet at the same point?
Answer 1b:

No. The perspective projection of two lines:

\[ L_1(t) = (x_1, y_1, z_1) + t(v_x, v_y, v_z), \quad L_2(s) = (x_2, y_2, z_2) + s(v_x, v_y, v_z) \]

Becomes:

\[ d\left(\frac{x_1 + tv_x}{z_1 + tv_z}, \frac{y_1 + tv_y}{z_1 + tv_z}\right), \quad d\left(\frac{x_2 + sv_x}{z_2 + sv_z}, \frac{y_2 + sv_y}{z_2 + sv_z}\right) \]

Both lines meet when \( s, t \rightarrow \text{Infinity} \), at \( \left(\frac{dv_x}{v_z}, \frac{dv_y}{v_z}\right) \)

(called the \textit{vanishing point}). All parallel lines meet at the same point.

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Question 2: Geometric Modelling

A bezier Curve is given:

\[ C(u) = \sum_{i=0}^{2} P_i B_i(u) \quad P_0 = (0,0) \quad P_1 = (0,1) \quad P_2 = (1,1) \]

- What is the value of the curve at \( u=0.5 \)?
- What is the degree of the curve?
- Given an “oracle” which compute \( C(u) \) for a given \( u \). How many samples are needed in order to reconstruct the entire curve?
- Represent \( C(u) \) as a curve of one degree higher than the current – as a function of new control points.
Answer 2

The Curve is:

\[ C(u) = P_0 u^2 + P_1 2u(1-u) + P_2 (1-u)^2 \]

- Value at 0.5:
  \[ C(0.5) = (0,0)0.5^2 + (0,1)0.52 \cdot (1-0.5) + (1,1)(1-0.5)^2 = \]
  \[ = (0,0) + (0,0.5) + (0.25,0.25) = (0.25,0.75) \]

- Degree of curve is 2
- To reconstruct a quadratic curve 3 samples are needed.
- The derivatives of the curve at the endpoints:
  \[ C'(0) = 2(P_1 - P_0), C'(1) = 2(P_2 - P_1) \]
Answer 2: Cont’d

- The new set of points (4 points for cubic curve) can now be deduced from the same derivatives at the endpoints (Q’s are new points):

\[ Q_0 = P_0 = (0, 0), \quad Q_3 = P_2 = (1, 1) \]

\[ 3(Q_3 - Q_2) = 2(P_2 - P_1), \quad 3(Q_1 - Q_0) = 2(P_1 - P_0) \]

\[ Q_2 = Q_3 - \frac{2}{3}(P_2 - P_1) = (1, 1) - \frac{2}{3}(1, 0) = \left( \frac{1}{3}, 1 \right) \]

\[ Q_1 = \frac{2}{3}(P_1 - P_0) + Q_0 = \frac{2}{3}(0, 1) + (0, 0) = (0, \frac{2}{3}) \]
Question 3: Texture Mapping

- Mipmaps - down-sampling a base texture by powers of two in both directions. A *Ripmap* set generates additional down-sampled textures by decoupling the down-sampling in each dimension.
  - For instance, an initial 4x4 texture will generate a ripmap set with sizes 4x2, 2x2, 2x4, 1x4, 1x2, 1x1, 2x1 and 4x1.
  - What is the memory consumption of the mipmaps\ripmaps of a symmetric texture of size n? (assume n is a power of 4)
  - When are ripmaps useful?
Answer 3

- Mipmap Memory Consumption:

\[ l = \log_4 n \]

\[ s_0 = n, s_1 = \frac{n}{4}, \ldots, s_i = \frac{n}{4^i}, s_l = 1 \]

\[ \sum_{i=0}^{l} s_i = n \sum_{i=0}^{l} \left( \frac{1}{4} \right)^i = n \frac{1 - \left( \frac{1}{4} \right)^{l+1}}{1 - \frac{1}{4}} = \frac{4}{3} n \left( 1 - \frac{1}{4n} \right) = \frac{4}{3} n - \frac{1}{3} \]

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Answer 3: Cont’d

- Ripmap memory consumption:
  - The two dimensions are independent!
  - For a $u$ side size of $m$ we get:

$$l = \log_2 m$$

$$\sum_{i=0}^{l} \frac{m}{2^i} \sum_{j=0}^{l} \frac{m}{2^j} = \frac{m \left(1 - \frac{1}{2^{l+1}}\right)}{1 - \frac{1}{2}} \cdot \frac{m \left(1 - \frac{1}{2^{l+1}}\right)}{1 - \frac{1}{2}} = 4n \left(1 - \frac{1}{2n}\right)^2$$

- Ripmap are good for when polygon are seen from a side view often. But the memory consumption is very large!

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Question 4: Antialiasing

- Given a 2D image
  - Do 8 rotations in 22.5 degrees result in the original image? Why?
  - Suggest a possible fix.

- Answer:
  - No, because filtering reduces the amount of information.
  - Rotating from the original image, instead of repeated rotations
    - Like in updating the 3D modelview matrix instead of the original coordinates!
Question 5: Color Theory

1. Assume the plane Z=0 is diffusive and white. In this scene, the following light sources exits:

Blue (0,0,1): At point (1,0,1)
Red (1,0,0): At point (3,2,1)

Where in the plane would we see pure magenta (1,0,1)?
Answer 5:

- The plane would be colored in magenta in the points equally distant from both light sources
  - In the bisector of the two projected locations of lights:
    - $z=0, y=-x+3$

- If we add Green $(0,1,0)$ at point $(-1,-1,1)$, where would we get white?
Answer 5b:

- In the meeting of all bisectors between any two lights
  - Also, the center of the unique circle that passes through them:

![Diagram of a circle with bisectors and a center point]
Question 6: Ray Tracing

- Answer yes/no to whether the following natural phenomena can be modeled with a simple backward ray-tracer, and explain your answer:
  - Reflections and refractions
  - Color bleeding (diffuse to diffuse surface reflection)
  - Shadows
  - Media rendering (materials in the air)
  - Visibility computation
  - Reflection of pure light (caustics)
Answer 6

- Reflections and refractions
  - Yes, the very reason for raytracing.

- Color bleeding (diffuse to diffuse surface reflection)
  - No, because the reflection is omnidirectional.

- Shadows
  - Also natural to raytracing.

- Media rendering (materials in the air)
  - Yes, integrating on a ray. Also the same for volume rendering

- Visibility computation
  - Natural to raytracing.

- Reflection of pure light (caustics)
  - No, only the reflection of one object unto another!